Univerität Basel

Herbsemester 2012

Master course A. Surroca - L. Paladino

Some topics on modular functions, elliptic functions and transcendence theory

Sheet of exercises n.5

5.1. Prove that $a_1 = (2\pi)^{12}$ in the Fourier expansion of the discriminant function Δ.

5.2. Let $\mathcal{M} = \{(\omega_1, \omega_2) \in \mathbb{C}^* | \omega_1/\omega_2 \in \mathcal{H}\}$. Consider two couples (ω_1, ω_2) and (ω_1', ω_2') in $\mathcal{M}$ and set $\Lambda := \mathbb{Z}_{\omega_1} \oplus \mathbb{Z}_{\omega_2}$, $\Lambda' := \mathbb{Z}_{\omega_1'} \oplus \mathbb{Z}_{\omega_2'}$. Prove that $\Lambda = \Lambda'$ if and only if there exists a matrix $\gamma = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ in $\mathrm{SL}_2(\mathbb{Z})$ such that

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} \omega_1 \\ \omega_2 \end{pmatrix} = \begin{pmatrix} \omega_1' \\ \omega_2' \end{pmatrix}.$$

5.3. Let F be a lattice function of weight k. For $\tau \in \mathcal{H}$, set $f(\tau) = F(\tau, 1)$. Prove that f is weakly modular of weight k.

5.4. Let f be weakly modular of weight k. For $(\omega_1, \omega_2) \in \mathcal{M}$, set $F(\omega_1, \omega_2) = \omega_2^{-2k} f(\omega_1/\omega_2)$ and for $\Lambda \in \mathcal{L}$, set $F(\Lambda) = F(\omega_1, \omega_2)$, where $\Lambda = \Lambda_{\omega_1, \omega_2}$. Prove that F is a lattice function of weight k.