Enablers and Inhibitors in Causal Justifications of Logic Programs

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UNIVERSIDADE DA CORUÑA

A Causal Semantics for Logic Programming
September 11th, 2015

Context: Causal Justifications

- Logic programming is an important paradigm for problem solving.
- Sometimes, is not enough to provide some conclusions, but we also are required to **provide explanations** for them.
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- A pair of **multi-valued** approaches with **algebraic constructions**:
  - **Why-not Provenance** [Damásio et al 2013]:
    oriented to debugging and explanation, well-founded semantics
  - **Causal Graphs** [Cabalar and Fandinno 14]:
    causes = proof graphs from (the positive part) of an ASP program

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Our proposal

We propose **Extended Causal Justifications** mixing both:

- **causal explanations** = proof graphs built with rule labels

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Our interest: representing **(relevant) causal knowledge** rather than debugging or providing all possible explanations

Motivation Example

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- If James Bond drinks drug `d`, he will have paralysis `p` unless has been administered an antidote `a`.
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\mathtt{p}: & \mathtt{p} & \leftarrow & \mathtt{d}, \mathtt{not}\ \mathtt{a} \\
\mathtt{a}: & \mathtt{a} & \leftarrow & \mathtt{not}\ \mathtt{h} \\
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- If James Bond drinks drug d, he will have paralysis p unless has been administered an antidote a.
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p: & p & \leftarrow & d, \text{not } a \\
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- Bond does not have paralysis, but **why not**?

Motivation Example

Example

- Suppose now that, that day, Bond was actually on a **holiday** with Vesper.

$$\begin{array}{lll} \mathtt{p}: & \mathtt{p} \leftarrow & \mathtt{d}, \mathtt{not\ a} \\ \mathtt{a}: & \mathtt{a} \leftarrow & \mathtt{not\ h} \\ \mathtt{d}: & \mathtt{d} & \\ \mathtt{h}: & \mathtt{h} & \end{array}$$

Example

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- **Counterfactuals**:
  - Had Le Chiffre not poured the drug d, Bond would not have paralysis d.
  - Had Bond not being on holiday h, he would not have paralysis d.

Motivation Example

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- Suppose now that, that day, Bond was actually on a **holiday** with Vesper.

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d :  d
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- **Counterfactuals**:
  - Had Le Chiffre not poured the drug *d*, Bond would not have paralysis *d*.
  - Had Bond not being on holiday *h*, he would not have paralysis *d*.

- **Causality**:
  - Is the drug *d* a cause of Bond's paralysis *d*? Yes!!
  - Is a holiday *h* a cause of Bond's paralysis *d*?

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- **Causality**:
  - Is the drug *d* a cause of Bond's paralysis *d*? Yes!!
  - Is a holiday *h* a cause of Bond's paralysis *d*? Controversial!!!

Motivation Example

- Is a holiday *h* a cause of Bond's paralysis *d*?
  - **Yes**: [Lewis73; Halpern and Pearl2001/2005]
  - No: [Hall2004/2007; Mauldlin2004]
  - May be: [Halpern and Hitchcock2011]

- Is a holiday h a cause of Bond's paralysis d?
  - **Yes**: [Lewis73; Halpern and Pearl2001/2005]
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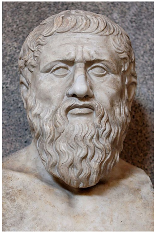

- Plato: "**distinguish the real cause** from that without which the cause would not be able to act as a cause."
  - The drug d is the real cause producing the paralysis.
  - The holiday h is an enabler of the cause.

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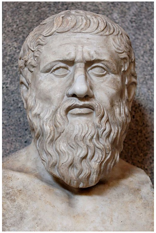

- Plato: "**distinguish the real cause** from that without which the cause would not be able to act as a cause."
  - The drug d is the real cause producing the paralysis.
  - The holiday h is an enabler of the cause.
- When not on holiday, the antidote a becomes an inhibitor of the potential cause (the drug) of Bond's paralysis.

In this work

- We propose a **multivalued semantics for LP**, Extended Causal Justifications (ECJ), that captures causes, enablers and inhibitors.
- **Generalizes**, under the well-founded semantics, both
  - **Causal Graph justifications** (CG): only captures real causes.
  - **Why-not Provenance** (WnP): only captures counterfactual dependence
- We also obtain a **formal comparation between CG and WnP**.

Outline

Positive programs

- Syntax: as usual plus an (optional) **rule label**

$$r_i : \quad H \leftarrow B_1, \ldots, B_m, \text{not } B_{m+1}, \ldots, \text{not } B_n$$

with H and B_j atoms. r_i can be a label $r_i = \ell$ or $t = 1$.

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with $\mathtt{H}$ and $\mathtt{B_j}$ atoms. $\mathtt{r_i}$ can be a label $\mathtt{r_i} = \ell$ or $\mathtt{t} = 1$.

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- We will assign the expression formed by rule labels $(\sim\mathtt{a} * \mathtt{d}) \cdot \mathtt{p}$ to the atom $\mathtt{p}$.
  - $\mathtt{d}$ (negated label) is an inhibitor of the drug $\mathtt{d}$.
  - '$\cdot$' captures the order of rule application: $\mathtt{p}$ would have been applied to $\mathtt{d}$ if $\mathtt{a}$ had not hold.

Causal Values

- **Causal terms** are expressions of the form

$$t \quad ::= \quad \ell \;\;|\;\; \prod S \;\;|\;\; \sum S \;\;|\;\; t_1 \cdot t_2 \;\;|\;\; {\sim} t_1$$

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- By 1 and 0 we denote the empty product $\prod \emptyset$ and sum $\sum \emptyset$, respectively.
- **Causal values** are the equivalence classes of causal terms under completely distributive (complete) lattice

Associativity	*Commutativity*	*Absorption*
$t + (u+w) = (t+u) + w$	$t + u = u + t$	$t = t + (t * u)$
$t * (u * w) = (t * u) * w$	$t * u = u * t$	$t = t * (t+u)$

Distributive	*Identity*	*Annihilator*
$t + (u * w) = (t+u) * (t+w)$	$t = t + 0$	$1 = 1 + t$
$t * (u+w) = (t * u) + (t * w)$	$t = t * 1$	$0 = 0 * t$

plus ...

Causal Values

- plus the following axioms for '·'

Associativity

$$t \cdot (u{\cdot}w) \;=\; (t{\cdot}u) \cdot w$$

Product Distributivity

$$\begin{aligned} c \cdot (d * e) &= (c \cdot d) * (c \cdot e) \\ (c * d) \cdot e &= (c \cdot e) * (d \cdot e) \end{aligned}$$

Identity

$$\begin{aligned} t &= t \cdot 1 \\ t &= 1 \cdot t \end{aligned}$$

Addition distributivity

$$\begin{aligned} t \cdot (u{+}w) &= (t{\cdot}u) + (t{\cdot}w) \\ (t + u) \cdot w &= (t{\cdot}w) + (u{\cdot}w) \end{aligned}$$

Absorption

$$\begin{aligned} t + u{\cdot}t{\cdot}w &= t \\ t * u{\cdot}t{\cdot}w &= u{\cdot}t{\cdot}w \end{aligned}$$

Annihilator

$$\begin{aligned} 0 &= t \cdot 0 \\ 0 &= 0 \cdot t \end{aligned}$$

Label idempotence

$$\ell \cdot \ell \;=\; \ell$$

Graph Representation

$$c \cdot d \cdot e \;=\; (c \cdot d) * (d \cdot e) \quad \text{with } d \neq 1$$

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- plus the following axioms for '$\sim$'

pseudo-compl.

$$t * {\sim} t = 0$$
$${\sim}{\sim}{\sim} t = {\sim} t$$

De Morgan

$${\sim}(t + u) = ({\sim} t * {\sim} u)$$
$${\sim}(t * u) = ({\sim} t + {\sim} u)$$

excluded middle

$${\sim} t + {\sim}{\sim} t = 1$$

appl. negation

$${\sim}(t \cdot u) = {\sim}(t * u)$$

- Every causal value can be represented by a term in **disjunctive normal form**
  - sum '$+$' is not in the scope of another connective: it separates **alternative causes**
  - negation '$\sim$' is only applied to labels or negated labels

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$$(\sim \mathtt{a} * \mathtt{d}) \cdot \mathtt{p} \; + \; (\sim\sim \mathtt{h} * \mathtt{d}) \cdot \mathtt{p}$$

- $\sim \mathtt{a}$ means that $\mathtt{a}$ is an **inhibitor** of $\mathtt{d}$.
- Negation is not classical, $\sim\sim \mathtt{h} \neq \mathtt{h}$ allows to **distinguish between an enabler $\mathtt{h}$ and the real cause $\mathtt{d}$** . . .

Definition (Causal model)

A **causal model** of P is an interpretation satisfying, for each rule:

$$\big(\ I(B_1) * \ldots * I(B_n) * I(\text{not } B_{m+1}) * \ldots * I(\text{not } B_n)\ \big) \cdot t \ \leq\ I(H)$$

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Theorem

A positive program has a least model, which can be computed by iteration of a direct consequences operator from the bottom interpretation.

Definition (Reduct)

The reduct P^J of a program P w.r.t. an interpretation J contains

$$r_i : \quad H \leftarrow B_1, \ldots, B_m, \; J(\text{not } B_{m+1}), \ldots, \; J(\text{not } B_n)$$

per each rule in P.

- By $\Gamma_P(J)$, we denote the least model I of P^J.
- $\Gamma_P(J)$ is antimonotonic and, thus $\Gamma_P^2(J)$ is monotonic.

Definition (Causal well-founded model)

The **causal well-founded model** $\mathbb{W}_P$ is a mapping such that

$$\mathbb{W}_P(A) \stackrel{def}{=} \mathrm{lfp}(\Gamma_P^2)(A)$$

$$\mathbb{W}_P(\text{not } A) \stackrel{def}{=} {\sim}\mathrm{gfp}(\Gamma_P^2)(A)$$

$$\mathbb{W}_P(\text{undef } A) \stackrel{def}{=} {\sim}\mathbb{W}_P(A) * {\sim}\mathbb{W}_P(\text{not } A)$$

- For instance, the least and greatest fixpoint of

$$\begin{array}{llll} \mathtt{p}: & \mathtt{p} \leftarrow \mathtt{d}, \mathtt{not}\ \mathtt{a} & \mathtt{d}: & \mathtt{d} \\ \mathtt{a}: & \mathtt{a} \leftarrow \mathtt{not}\ \mathtt{h} & \mathtt{h}: & \mathtt{h} \end{array}$$

coincide and satisfy

$$\mathrm{lfp}(\Gamma_{\mathtt{P}}^2)(\mathtt{a}) \quad = \quad {\sim}\mathtt{h}\cdot\mathtt{a} \qquad\qquad \mathrm{lfp}(\Gamma_{\mathtt{P}}^2)(\mathtt{p}) \quad = \quad ({\sim}\mathtt{a} * \mathtt{d})\cdot\mathtt{p} + ({\sim\sim}\mathtt{h} * \mathtt{d})\cdot\mathtt{p}$$

- Each addend is a justification: ${\sim}\mathtt{h}\cdot\mathtt{a}$, $({\sim}\mathtt{a} * \mathtt{d})\cdot\mathtt{p}$ and $({\sim\sim}\mathtt{h} * \mathtt{d})\cdot\mathtt{p}$.

Well-founded causal model

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  - The antidote a has not been administered because Bond was on a holiday h.

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- Each addend is a justification: ${\sim}\mathtt{h}{\cdot}\mathtt{a}$, $({\sim}\mathtt{a} * \mathtt{d}){\cdot}\mathtt{p}$ and $({\sim\sim}\mathtt{h} * \mathtt{d}){\cdot}\mathtt{p}$.
- If a justification contains a negated label (odd num. of times) is said to be inhibited.
  - The antidote $\mathtt{a}$ has not been administered because Bond was on a holiday $\mathtt{h}$.
- Otherwise, a justification is said to be enabled.
  - Bond is paralyzed because $\mathtt{h}$ has enabled $\mathtt{d}$ to cause $\mathtt{p}$: $({\sim\sim}\mathtt{h} * \mathtt{d}){\cdot}\mathtt{p}$.

- The axiom $\sim(t \cdot u) = \sim(t * u)$ allow us to break justifications into inhibited and enabled:

$$
\begin{aligned}
(\sim(\sim h \cdot a) * d) \cdot p &= (\sim(\sim h * a) * d) \cdot p \\
&= ((\sim\sim h + \sim a) * d) \cdot p \\
&= (\sim\sim h * d) \cdot p \; + \; (\sim a * d) \cdot p
\end{aligned}
$$

Theorem

An atom is true, false of undefined in the standard well-founded model iff there is some ***enabled justification*** *for it.*

Theorem

Inhibited justifications become enabled justifications when the inhibitors are removed.

- For instance, $\sim\mathtt{h}\cdot\mathtt{a}$ is an inhibited justification for $\mathtt{a}$ in

$\mathtt{p}: \ \mathtt{p} \leftarrow \mathtt{d},\ \mathtt{not}\ \mathtt{a}$	$\mathtt{d}: \ \mathtt{d}$
$\mathtt{a}: \ \mathtt{a} \leftarrow \mathtt{not}\ \mathtt{h}$	$\mathtt{h}: \ \mathtt{h}$

and rule $\mathtt{a}$ is an enabled justification for $\mathtt{a}$ in

$\mathtt{p}: \ \mathtt{p} \leftarrow \mathtt{d},\ \mathtt{not}\ \mathtt{a}$	$\mathtt{d}: \ \mathtt{d}$
$\mathtt{a}: \ \mathtt{a} \leftarrow \mathtt{not}\ \mathtt{h}$	~~$\mathtt{h}: \ \mathtt{h}$~~

Undefined literals

- Consider the usual cycle

$$\mathtt{r_1}: \quad \mathtt{a} \leftarrow \mathtt{not\ b} \qquad\qquad \mathtt{r_2}: \quad \mathtt{b} \leftarrow \mathtt{not\ a}$$

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$$\mathbb{W}_P(\mathtt{a}) = {\sim}\mathtt{r}_2 \cdot \mathtt{r}_1 \qquad \mathbb{W}_P(\mathtt{not\ a}) = {\sim}\mathtt{r}_1$$

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$$\mathbb{W}_P(\mathtt{a}) = {\sim}\mathtt{r}_2 \cdot \mathtt{r}_1 \qquad \mathbb{W}_P(\mathtt{not\ a}) = {\sim}\mathtt{r}_1$$

- $\mathtt{a}$ is not true because of $\mathtt{r}_2$ and it is not false because of $\mathtt{r}_1$.

Undefined literals

- Consider the usual cycle

$$r_1: \quad a \leftarrow \text{not } b \qquad\qquad r_2: \quad b \leftarrow \text{not } a$$

- The least and greatest fixpoint do not coincide.

$$\text{lfp}(\Gamma_P^2)(a) = \sim r_2 \cdot r_1 \qquad \text{gfp}(\Gamma_P^2)(a) = r_1$$

$$\mathbb{W}_P(a) = \sim r_2 \cdot r_1 \qquad \mathbb{W}_P(\text{not } a) = \sim r_1$$

- a is not true because of r_2 and it is not false because of r_1.
- a is undefined because of rules r_1 and r_2.

$$\begin{aligned} \mathbb{W}_P(\text{undef } a) &= \sim\mathbb{W}_P(a) * \mathbb{W}_P(\text{not } a) \\ &= \sim(\sim r_2 \cdot r_1) * \sim\sim r_1 = \sim\sim r_1 * \sim\sim r_2 \end{aligned}$$

Outline

Causal Graph Justifications

- Causal Graph Justifications (CG) is an extension of the **stable model semantics** whereas ECJ is an extension of the **well-founded semantics**.

- Causal Graph Justifications (CG) is an extension of the **stable model semantics** whereas ECJ is an extension of the **well-founded semantics**.
- CG does **not capture enablers** nor **inhibited justifications**.

$$(\sim a * d) \cdot p \xrightarrow{\lambda^c} 0 \qquad\qquad (\sim\sim h * d) \cdot p \xrightarrow{\lambda^c} d \cdot p$$

- ECJ justifications can be mapped into CG justifications.
  - Removing all inhibited justifications and
  - Removing all enablers for the remaining ones.

CG stable models can be related to the Γ_P^2 fixpoints.

Theorem

For any enabled justification, there is a CG-justification w.r.t all stable models obtained by removing all enablers.

- The converse does not hold in general as happened with the standard well-founded and stable model semantics.

$r_1: \ a \leftarrow not\ b \qquad r_2: \ b \leftarrow not\ a, not\ c \qquad r_4: \ d \leftarrow b, not\ d$

$r_3: \ c \leftarrow a \qquad c: \ c$

- Two-valued standard well-founded model $\{a, c\}$. $\mathbb{W}_P(c) = c$. Unique standard stable model $\{a, c\}$. $I(c) = c + r_1 \cdot r_3$.

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$$(\sim\mathtt{a} * \mathtt{d}) \cdot \mathtt{p} \xrightarrow{\lambda^p} \neg\mathtt{a} \wedge \mathtt{d} \wedge \mathtt{p} \qquad (\sim\sim\mathtt{h} * \mathtt{d}) \cdot \mathtt{p} \xrightarrow{\lambda^p} \mathtt{h} \wedge \mathtt{d} \wedge \mathtt{p} \qquad \mathtt{d} \cdot \mathtt{p} \xrightarrow{\lambda^p} \mathit{not}(\mathtt{a}) \wedge \mathtt{d} \wedge \mathtt{p}$$

- ECJ justifications can be mapped into WnP justifications.
  - Replacing both, '$*$' and '$\cdot$', by '$\wedge$'.
  - Replacing double negated labels by positive ones $\sim\sim\mathtt{h} \Longrightarrow \mathtt{h}$
  - Adding $\mathit{not}(\mathtt{a})$ for each negative literal `not a` in the body of some rule in the justification.

Why-not Provenance Justifications

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- WnP may contain more justifications that we call **hypothetical**.
  - Suppose that Le Chiffre did not pour the drug and Bond is not on holidays.

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$$\texttt{a}: \ \texttt{a} \leftarrow \texttt{not h}$$

- Clearly, Bond is not paralyzed, but there is an hypothetical WnP justification $\neg \mathit{not}(\texttt{d}) \wedge \neg \mathit{not}(\texttt{h}) \wedge \mathit{not}(\texttt{a}) \wedge \texttt{p}$ meaning that p would have been true if we had added facts d and h to the program while not adding fact a.

Theorem

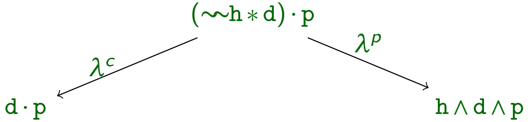
For any non-hypothetical WnP justification, there is a corresponding ECJ justification, and vice-versa.

- Hypothetical WnP justifications can be captured augmenting the program with facts

$$\sim\!\mathit{not}(\mathtt{a}) : \quad \mathtt{a}$$

for every atom a which is not already a fact.

- ECJ justifications can be mapped into both, CG and WnP Justifications.



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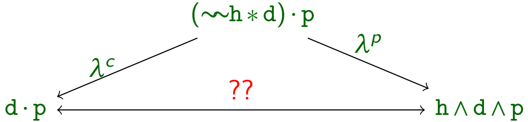

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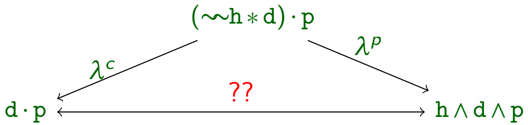

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For any non-hypothetical and enabled WnP-justification, there is some CG-justification, w.r.t. to all causal stable models, such that the former contains all the vertices of the later (may contain more).

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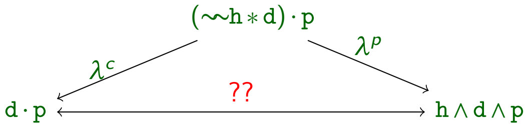

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Theorem

For any non-hypothetical and enabled WnP-justification, there is some CG-justification, w.r.t. to all causal stable models, such that the former contains all the vertices of the later (may contain more).

- As usual the converse does not hold.

Conclusions

- A **multivalued** extension of **well-founded semantics** that captures inhibitors, enablers and causes by introducing '∼' in the CG algebra.
- Allows distinguishing between **enablers** '∼∼a' and real **causes** 'a'.
- The existence of enabled justifications is a **sufficient and necessary condition** for the truth value of an atom.
- It **captures both WnP and CJ** justification.
- We established a **formal relation between WnP and CJ**.

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- We established a **formal relation between WnP and CJ**.

Ongoing work

- Incorporate enablers and inhibitors to the **stable model semantics**.
- **Deriving new conclusions** from the cause-effect relations (ASPOCP 2015)
- **Non deterministic** causal laws: disjunctive and probabilistic LP

Enablers and Inhibitors in Causal Justifications of Logic Programs

Pedro Cabalar and Jorge Fandinno

Thanks for your attention!

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LPNMR'15
Lexington, KY, USA

- Programs with odd negative cycles may have standard stable model, but no Γ_P fixpoint

$$\begin{aligned} r_1 : p &\leftarrow \\ r_2 : p &\leftarrow \ \text{not}\ p \end{aligned}$$

- ... although they have Γ_P^2 fixpoints.

$$\text{lfp}(\Gamma_P^2)(p) \ = \ r_1 \qquad \qquad \text{gfp}(\Gamma_P^2)(p) \ = \ r_1 + {\sim} r_1 \cdot r_2$$

- p is clearly true because of r_1, but what about r_2?