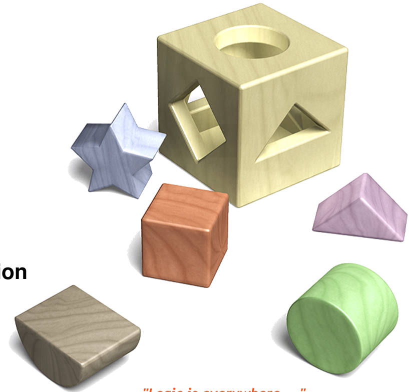
A New Computational Logic Approach to Reason with Conditionals

Emmanuelle-Anna Dietz and Steffen Hölldobler
International Center for Computational Logic
Technische Universität Dresden
Germany

- Introduction
- Weak Completion Semantics
- Indicative Conditionals
- Minimal Revision Followed by Abduction
- Subjunctive Conditionals
- Future Work



"Logic is everywhere ..."

The Firing Squad Example

- **If the court orders an execution, then the captain will give the signal upon which rifleman *A* and *B* will shoot the prisoner; consequently, the prisoner will be dead** Pearl 2000

The Firing Squad Example

- **If the court orders an execution, then the captain will give the signal upon which rifleman *A* and *B* will shoot the prisoner; consequently, the prisoner will be dead** Pearl 2000
- **We assume that**
  - **the court's decision is *unknown***
  - **both riflemen are accurate, alert and law-abiding**
  - **the prisoner is unlikely to die from any other causes**

The Firing Squad Example

- **If the court orders an execution, then the captain will give the signal upon which rifleman *A* and *B* will shoot the prisoner; consequently, the prisoner will be dead** Pearl 2000
- **We assume that**
  - **the court's decision is *unknown***
  - **both riflemen are accurate, alert and law-abiding**
  - **the prisoner is unlikely to die from any other causes**
- **Please evaluate the following conditionals (*true*, *false*, *unknown*)**

The Firing Squad Example

- **If the court orders an execution, then the captain will give the signal upon which rifleman *A* and *B* will shoot the prisoner; consequently, the prisoner will be dead** Pearl 2000
- **We assume that**
  - **the court's decision is *unknown***
  - **both riflemen are accurate, alert and law-abiding**
  - **the prisoner is unlikely to die from any other causes**
- **Please evaluate the following conditionals (*true*, *false*, *unknown*)**
  - **If the prisoner is alive, then the captain did not signal**

The Firing Squad Example

- If the court orders an execution, then the captain will give the signal upon which rifleman *A* and *B* will shoot the prisoner; consequently, the prisoner will be dead — Pearl 2000
- We assume that
  - the court's decision is *unknown*
  - both riflemen are accurate, alert and law-abiding
  - the prisoner is unlikely to die from any other causes
- Please evaluate the following conditionals (*true*, *false*, *unknown*)
  - If rifleman A shot, then rifleman B shot as well

The Firing Squad Example

- **If the court orders an execution, then the captain will give the signal upon which rifleman *A* and *B* will shoot the prisoner; consequently, the prisoner will be dead** Pearl 2000
- **We assume that**
  - **the court's decision is *unknown***
  - **both riflemen are accurate, alert and law-abiding**
  - **the prisoner is unlikely to die from any other causes**
- **Please evaluate the following conditionals (*true*, *false*, *unknown*)**
  - **If the captain gave no signal and rifleman A decides to shoot, then the court did not order an execution**

Introduction

- **The weak completion semantics (WCS)**
  - **is a new cognitive theory**
  - **is based on ideas presented in** Stenning, van Lambalgen 2008
  - **is mathematically sound** H., Kencana Ramli 2009
  - **has been successfully applied to model–among others– the suppression task, the selection task, and the belief bias effect**
    Dietz, H., Ragni 2012, Dietz, H., Ragni 2013, Pereira, Dietz, H. 2014

Introduction

- **The weak completion semantics (WCS)**
  - **is a new cognitive theory**
  - **is based on ideas presented in** Stenning, van Lambalgen 2008
  - **is mathematically sound** H., Kencana Ramli 2009
  - **has been successfully applied to model–among others–
    the suppression task, the selection task, and the belief bias effect**
    Dietz, H., Ragni 2012, Dietz, H., Ragni 2013, Pereira, Dietz, H. 2014
- **Now, we want to apply WCS to reason about conditionals**

Logic Programs and their Weak Completion

- **Let $\mathcal{P}$ be a ground logic program**
- **Let $\mathcal{S}$ be a finite and consistent set of ground literals**

Logic Programs and their Weak Completion

- Let $\mathcal{P}$ be a ground logic program
- Let $\mathcal{S}$ be a finite and consistent set of ground literals
- $def(\mathcal{S}, \mathcal{P}) = \{A \leftarrow body \in \mathcal{P} \mid A \in \mathcal{S} \vee \neg A \in \mathcal{S}\}$

Logic Programs and their Weak Completion

- **Let $\mathcal{P}$ be a ground logic program**
- **Let $\mathcal{S}$ be a finite and consistent set of ground literals**
- $\mathit{def}(\mathcal{S}, \mathcal{P}) = \{A \leftarrow \mathit{body} \in \mathcal{P} \mid A \in \mathcal{S} \vee \neg A \in \mathcal{S}\}$
- **Weak completion** $\mathit{wc}\mathcal{P}$ **=** **completion of** $\mathcal{P} \quad \setminus \quad \{A \leftrightarrow \bot \mid \mathit{def}(A, \mathcal{P}) = \emptyset\}$

Logic Programs and their Weak Completion

- Let $\mathcal{P}$ be a ground logic program
- Let $\mathcal{S}$ be a finite and consistent set of ground literals
- $def(\mathcal{S}, \mathcal{P}) = \{A \leftarrow body \in \mathcal{P} \mid A \in \mathcal{S} \vee \neg A \in \mathcal{S}\}$
- Weak completion $wc\mathcal{P}$ = completion of $\mathcal{P}$ $\setminus$ $\{A \leftrightarrow \bot \mid def(A, \mathcal{P}) = \emptyset\}$

$$wc\{a \leftarrow b,\ a \leftarrow c,\ c \leftarrow \bot\} \quad = \quad \{a \leftrightarrow b \vee c,\ c \leftrightarrow \bot\}$$

Logic Programs and their Weak Completion

- Let $\mathcal{P}$ be a ground logic program
- Let $\mathcal{S}$ be a finite and consistent set of ground literals
- $def(\mathcal{S}, \mathcal{P}) = \{A \leftarrow body \in \mathcal{P} \mid A \in \mathcal{S} \vee \neg A \in \mathcal{S}\}$
- Weak completion $wc\mathcal{P}$ = completion of $\mathcal{P}$ $\setminus$ $\{A \leftrightarrow \bot \mid def(A, \mathcal{P}) = \emptyset\}$

$$wc\{a \leftarrow b,\ a \leftarrow c,\ c \leftarrow \bot\} \quad = \quad \{a \leftrightarrow b \vee c,\ c \leftrightarrow \bot\}$$
$$wc\{c \leftarrow \top,\ c \leftarrow \bot\} \quad = \quad \{c \leftrightarrow \top \vee \bot\}$$

Weak Completion Semantics

- H., Kencana Ramli: Logic Programs under Three-Valued Łukasiewicz's Semantics
  In: Hill, Warren (eds), Logic Programming, LNCS 5649, 464-478: 2009
- **We consider the three-valued Łukasiewicz logic** Łukasiewicz 1920
  - $\mathbf{U} \leftarrow \mathbf{U} = \top$ (compared to $\mathsf{U} \leftarrow \mathsf{U} = \mathsf{U}$ under Kripke-Kleene logic)

Weak Completion Semantics

- H., Kencana Ramli: Logic Programs under Three-Valued Łukasiewicz's Semantics
  In: Hill, Warren (eds), Logic Programming, LNCS 5649, 464-478: 2009
- **We consider the three-valued Łukasiewicz logic** Łukasiewicz 1920
  - $\mathbf{U} \leftarrow \mathbf{U} = \top$ (compared to $\mathsf{U} \leftarrow \mathsf{U} = \mathsf{U}$ under Kripke-Kleene logic)
- **Each weakly completed program admits a least model $\mathcal{M}_{\mathcal{P}}$ under Ł-logic**

Weak Completion Semantics

- H., Kencana Ramli: Logic Programs under Three-Valued Łukasiewicz's Semantics
  In: Hill, Warren (eds), Logic Programming, LNCS 5649, 464-478: 2009

- **We consider the three-valued Łukasiewicz logic** Łukasiewicz 1920
  - $\mathsf{U} \leftarrow \mathsf{U} = \top$ (compared to $\mathsf{U} \leftarrow \mathsf{U} = \mathsf{U}$ under Kripke-Kleene logic)

- **Each weakly completed program admits a least model $\mathcal{M}_\mathcal{P}$ under Ł-logic**

- **$\mathcal{M}_\mathcal{P}$ is the least fixed point of $\Phi_\mathcal{P}(I) = \langle J^\top, J^\bot \rangle$, where**

$$J^\top = \{A \mid A \leftarrow body \in \mathcal{P} \quad \text{and} \quad I(body) = \top\}$$
$$J^\bot = \{A \mid def(A, \mathcal{P}) \neq \emptyset \quad \text{and} \quad I(body) = \bot \text{ for all } A \leftarrow body \in def(A, \mathcal{P})\}$$

($\Phi_\mathcal{P}$ is due to Stenning, vanLambalgen 2008)

Weak Completion Semantics

- H., Kencana Ramli: Logic Programs under Three-Valued Łukasiewicz's Semantics
  In: Hill, Warren (eds), Logic Programming, LNCS 5649, 464-478: 2009

- **We consider the three-valued Łukasiewicz logic** Łukasiewicz 1920
  - $\mathsf{U} \leftarrow \mathsf{U} = \top$ (compared to $\mathsf{U} \leftarrow \mathsf{U} = \mathsf{U}$ under Kripke-Kleene logic)

- **Each weakly completed program admits a least model $\mathcal{M}_\mathcal{P}$ under Ł-logic**

- **$\mathcal{M}_\mathcal{P}$ is the least fixed point of $\Phi_\mathcal{P}(I) = \langle J^\top, J^\bot \rangle$, where**

$$J^\top = \{A \mid A \leftarrow body \in \mathcal{P} \text{ and } I(body) = \top\}$$
$$J^\bot = \{A \mid def(A, \mathcal{P}) \neq \emptyset \text{ and } I(body) = \bot \text{ for all } A \leftarrow body \in def(A, \mathcal{P})\}$$

($\Phi_\mathcal{P}$ is due to Stenning, vanLambalgen 2008)

- $\mathcal{P} \models_{wcs} F$ **iff** $\mathcal{M}_\mathcal{P}(F) = \top$

Weak Completion Semantics

- H., Kencana Ramli: Logic Programs under Three-Valued Łukasiewicz's Semantics
  In: Hill, Warren (eds), Logic Programming, LNCS 5649, 464-478: 2009
- **We consider the three-valued Łukasiewicz logic** Łukasiewicz 1920
  - $\mathsf{U} \leftarrow \mathsf{U} = \top$ (compared to $\mathsf{U} \leftarrow \mathsf{U} = \mathsf{U}$ under Kripke-Kleene logic)
- **Each weakly completed program admits a least model $\mathcal{M}_\mathcal{P}$ under Ł-logic**
- **$\mathcal{M}_\mathcal{P}$ is the least fixed point of $\Phi_\mathcal{P}(I) = \langle J^\top, J^\bot \rangle$, where**

$$J^\top = \{A \mid A \leftarrow body \in \mathcal{P} \ \text{ and } \ I(body) = \top\}$$
$$J^\bot = \{A \mid def(A, \mathcal{P}) \neq \emptyset \ \text{ and } \ I(body) = \bot \text{ for all } A \leftarrow body \in def(A, \mathcal{P})\}$$

($\Phi_\mathcal{P}$ is due to Stenning, vanLambalgen 2008)

- $\mathcal{P} \models_{wcs} F$ **iff** $\mathcal{M}_\mathcal{P}(F) = \top$

$$\{a \leftarrow b,\ a \leftarrow c,\ c \leftarrow \bot\} \quad \not\models_{wcs} \quad a \vee \neg a$$

Weak Completion Semantics

- H., Kencana Ramli: Logic Programs under Three-Valued Łukasiewicz's Semantics In: Hill, Warren (eds), Logic Programming, LNCS 5649, 464-478: 2009
- **We consider the three-valued Łukasiewicz logic** Łukasiewicz 1920
  - $\mathsf{U} \leftarrow \mathsf{U} = \top$ (compared to $\mathsf{U} \leftarrow \mathsf{U} = \mathsf{U}$ under Kripke-Kleene logic)
- **Each weakly completed program admits a least model $\mathcal{M}_\mathcal{P}$ under Ł-logic**
- **$\mathcal{M}_\mathcal{P}$ is the least fixed point of $\Phi_\mathcal{P}(I) = \langle J^\top, J^\bot \rangle$, where**

$$J^\top = \{A \mid A \leftarrow body \in \mathcal{P} \ \text{ and } \ I(body) = \top\}$$
$$J^\bot = \{A \mid def(A,\mathcal{P}) \neq \emptyset \ \text{ and } \ I(body) = \bot \text{ for all } A \leftarrow body \in def(A,\mathcal{P})\}$$

($\Phi_\mathcal{P}$ is due to Stenning, vanLambalgen 2008)

- $\mathcal{P} \models_{wcs} F$ **iff** $\mathcal{M}_\mathcal{P}(F) = \top$

$$\{a \leftarrow b,\ a \leftarrow c,\ c \leftarrow \bot\} \quad \not\models_{wcs} \quad a \vee \neg a$$
$$\{c \leftarrow \top,\ c \leftarrow \bot\} \quad \models_{wcs} \quad c$$

Abduction

- Consider the abductive framework $\langle \mathcal{P}, \mathcal{A}_\mathcal{P}, \mathcal{IC}, \models_{wcs} \rangle$, where
  - $\mathcal{A}_\mathcal{P} = \bigcup_{\{A \mid def(A,\mathcal{P})=\emptyset\}} \{A \leftarrow \top,\ A \leftarrow \bot\}$ is the set of abducibles
  - $\mathcal{IC}$ is a finite set of integrity constraints

Abduction

- Consider the abductive framework $\langle \mathcal{P}, \mathcal{A}_\mathcal{P}, \mathcal{IC}, \models_{wcs} \rangle$, where
  - $\mathcal{A}_\mathcal{P} = \bigcup_{\{A \mid def(A,\mathcal{P})=\emptyset\}} \{A \leftarrow \top,\ A \leftarrow \bot\}$ is the set of abducibles
  - $\mathcal{IC}$ is a finite set of integrity constraints
- An observation $\mathcal{O}$ is a set of ground literals
  - $\mathcal{O}$ is explainable in $\langle \mathcal{P}, \mathcal{A}_\mathcal{P}, \mathcal{IC}, \models_{wcs} \rangle$
    iff there exists a minimal $\mathcal{E} \subseteq \mathcal{A}_\mathcal{P}$ called explanation such that
    $\mathcal{M}_{\mathcal{P}\cup\mathcal{E}}$ satisfies $\mathcal{IC}$ and $\mathcal{P} \cup \mathcal{E} \models_{wcs} \mathcal{O}$

Revision

- $rev(\mathcal{P}, \mathcal{S}) = (\mathcal{P} \setminus def(\mathcal{S}, \mathcal{P})) \cup \{A \leftarrow \top \mid A \in \mathcal{S}\} \cup \{A \leftarrow \bot \mid \neg A \in \mathcal{S}\}$

Revision

- $rev(\mathcal{P}, \mathcal{S}) = (\mathcal{P} \setminus def(\mathcal{S}, \mathcal{P})) \cup \{A \leftarrow \top \mid A \in \mathcal{S}\} \cup \{A \leftarrow \bot \mid \neg A \in \mathcal{S}\}$
- Properties
  - *rev* is non-monotonic in general
  - *rev* is monotonic, i.e., $\mathcal{M}_{\mathcal{P}} \subseteq \mathcal{M}_{rev(\mathcal{P},\mathcal{S})}$, if $\mathcal{M}_{\mathcal{P}}(L) = U$ for all $L \in \mathcal{S}$
  - $\mathcal{M}_{rev(\mathcal{P},\mathcal{S})}(\mathcal{S}) = \top$

Conditionals

- **Conditionals** **are statements of the form** ***if condition then consequence***

Conditionals

- **Conditionals** are statements of the form *if condition then consequence*
- **Indicative conditionals** are conditionals
  - whose condition may or may not be *true*
  - whose consequence may or may not be *true*
  - but the consequence is asserted to be *true* if the condition is *true*

Conditionals

- **Conditionals** are statements of the form ***if condition then consequence***
- **Indicative conditionals** are conditionals
  - whose condition may or may not be *true*
  - whose consequence may or may not be *true*
  - but the consequence is asserted to be *true* if the condition is *true*
- **Subjunctive conditionals** are conditionals
  - whose condition is *false*
  - whose consequence may or may not be *true*
  - but in the counterfactual circumstance of the condition being *true*, the consequence is asserted to be *true* as well

Indicative Conditionals

- In the sequel, let *cond*$(\mathcal{C}, \mathcal{D})$ be an indicative conditional, where
  - Condition $\mathcal{C}$ and consequence $\mathcal{D}$
    are finite and consistent sets of ground literals

Indicative Conditionals

- In the sequel, let $cond(\mathcal{C}, \mathcal{D})$ be an indicative conditional, where
  - Condition $\mathcal{C}$ and consequence $\mathcal{D}$
    are finite and consistent sets of ground literals
- Conditionals are evaluated wrt a given $\mathcal{P}$ and $\mathcal{IC}$
  - We assume that $\mathcal{M}_{\mathcal{P}}$ satisfies $\mathcal{IC}$

Evaluating Indicative Conditionals – Our Approach

- **Given $\mathcal{P}$, $\mathcal{IC}$, and *cond*$(\mathcal{C}, \mathcal{D})$**

Evaluating Indicative Conditionals – Our Approach

- **Given $\mathcal{P}$, $\mathcal{IC}$, and *cond*$(\mathcal{C}, \mathcal{D})$**
  - **If $\mathcal{M}_{\mathcal{P}}(\mathcal{C}) = \top$ and $\mathcal{M}_{\mathcal{P}}(\mathcal{D}) = \top$ then *cond*$(\mathcal{C}, \mathcal{D})$ is *true***

Evaluating Indicative Conditionals – Our Approach

- **Given $\mathcal{P}$, $\mathcal{IC}$, and *cond*$(\mathcal{C}, \mathcal{D})$**
  - **If $\mathcal{M}_{\mathcal{P}}(\mathcal{C}) = \top$ and $\mathcal{M}_{\mathcal{P}}(\mathcal{D}) = \top$ then *cond*$(\mathcal{C}, \mathcal{D})$ is *true***
  - **If $\mathcal{M}_{\mathcal{P}}(\mathcal{C}) = \top$ and $\mathcal{M}_{\mathcal{P}}(\mathcal{D}) = \bot$ then *cond*$(\mathcal{C}, \mathcal{D})$ is *false***

Evaluating Indicative Conditionals – Our Approach

- **Given $\mathcal{P}$, $\mathcal{IC}$, and *cond*$(\mathcal{C}, \mathcal{D})$**
  - **If $\mathcal{M}_{\mathcal{P}}(\mathcal{C}) = \top$ and $\mathcal{M}_{\mathcal{P}}(\mathcal{D}) = \top$ then *cond*$(\mathcal{C}, \mathcal{D})$ is *true***
  - **If $\mathcal{M}_{\mathcal{P}}(\mathcal{C}) = \top$ and $\mathcal{M}_{\mathcal{P}}(\mathcal{D}) = \bot$ then *cond*$(\mathcal{C}, \mathcal{D})$ is *false***
  - **If $\mathcal{M}_{\mathcal{P}}(\mathcal{C}) = \top$ and $\mathcal{M}_{\mathcal{P}}(\mathcal{D}) = \mathsf{U}$ then *cond*$(\mathcal{C}, \mathcal{D})$ is *unknown***

Evaluating Indicative Conditionals – Our Approach

- **Given $\mathcal{P}$, $\mathcal{IC}$, and *cond*$(\mathcal{C}, \mathcal{D})$**
  - **If $\mathcal{M}_\mathcal{P}(\mathcal{C}) = \top$ and $\mathcal{M}_\mathcal{P}(\mathcal{D}) = \top$ then *cond*$(\mathcal{C}, \mathcal{D})$ is *true***
  - **If $\mathcal{M}_\mathcal{P}(\mathcal{C}) = \top$ and $\mathcal{M}_\mathcal{P}(\mathcal{D}) = \bot$ then *cond*$(\mathcal{C}, \mathcal{D})$ is *false***
  - **If $\mathcal{M}_\mathcal{P}(\mathcal{C}) = \top$ and $\mathcal{M}_\mathcal{P}(\mathcal{D}) = \mathrm{U}$ then *cond*$(\mathcal{C}, \mathcal{D})$ is *unknown***
  - **If $\mathcal{M}_\mathcal{P}(\mathcal{C}) = \bot$ then *cond*$(\mathcal{C}, \mathcal{D})$ is *vacuous***

Evaluating Indicative Conditionals – Our Approach

- Given $\mathcal{P}$, $\mathcal{IC}$, and $\mathit{cond}(\mathcal{C}, \mathcal{D})$
  - If $\mathcal{M}_{\mathcal{P}}(\mathcal{C}) = \top$ and $\mathcal{M}_{\mathcal{P}}(\mathcal{D}) = \top$ then $\mathit{cond}(\mathcal{C}, \mathcal{D})$ is *true*
  - If $\mathcal{M}_{\mathcal{P}}(\mathcal{C}) = \top$ and $\mathcal{M}_{\mathcal{P}}(\mathcal{D}) = \bot$ then $\mathit{cond}(\mathcal{C}, \mathcal{D})$ is *false*
  - If $\mathcal{M}_{\mathcal{P}}(\mathcal{C}) = \top$ and $\mathcal{M}_{\mathcal{P}}(\mathcal{D}) = \mathsf{U}$ then $\mathit{cond}(\mathcal{C}, \mathcal{D})$ is *unknown*
  - If $\mathcal{M}_{\mathcal{P}}(\mathcal{C}) = \bot$ then $\mathit{cond}(\mathcal{C}, \mathcal{D})$ is *vacuous*
  - If $\mathcal{M}_{\mathcal{P}}(\mathcal{C}) = \mathsf{U}$ then evaluate $\mathit{cond}(\mathcal{C}, \mathcal{D})$ with respect to $\mathcal{M}_{\mathcal{P}'}$, where
    - $\mathcal{P}' = \mathit{rev}(\mathcal{P}, \mathcal{S}) \cup \mathcal{E}$,
    - $\mathcal{S}$ is a smallest subset of $\mathcal{C}$ and
      $\mathcal{E} \subseteq \mathcal{A}_{\mathit{rev}(\mathcal{P},\mathcal{S})}$ is an explanation for $\mathcal{C} \setminus \mathcal{S}$ such that
      $\mathcal{P}' \models_{\mathit{wcs}} \mathcal{C}$ and $\mathcal{M}_{\mathcal{P}'}$ satisfies $\mathcal{IC}$

Evaluating Indicative Conditionals – Our Approach

- Given $\mathcal{P}$, $\mathcal{IC}$, and $cond(\mathcal{C}, \mathcal{D})$
  - If $\mathcal{M}_\mathcal{P}(\mathcal{C}) = \top$ and $\mathcal{M}_\mathcal{P}(\mathcal{D}) = \top$ then $cond(\mathcal{C}, \mathcal{D})$ is *true*
  - If $\mathcal{M}_\mathcal{P}(\mathcal{C}) = \top$ and $\mathcal{M}_\mathcal{P}(\mathcal{D}) = \bot$ then $cond(\mathcal{C}, \mathcal{D})$ is *false*
  - If $\mathcal{M}_\mathcal{P}(\mathcal{C}) = \top$ and $\mathcal{M}_\mathcal{P}(\mathcal{D}) = \mathsf{U}$ then $cond(\mathcal{C}, \mathcal{D})$ is *unknown*
  - If $\mathcal{M}_\mathcal{P}(\mathcal{C}) = \bot$ then $cond(\mathcal{C}, \mathcal{D})$ is *vacuous*
  - If $\mathcal{M}_\mathcal{P}(\mathcal{C}) = \mathsf{U}$ then evaluate $cond(\mathcal{C}, \mathcal{D})$ with respect to $\mathcal{M}_{\mathcal{P}'}$, where
    - $\mathcal{P}' = rev(\mathcal{P}, \mathcal{S}) \cup \mathcal{E}$,
    - $\mathcal{S}$ is a smallest subset of $\mathcal{C}$ and
      $\mathcal{E} \subseteq \mathcal{A}_{rev(\mathcal{P},\mathcal{S})}$ is an explanation for $\mathcal{C} \setminus \mathcal{S}$ such that
      $\mathcal{P}' \models_{wcs} \mathcal{C}$ and $\mathcal{M}_{\mathcal{P}'}$ satisfies $\mathcal{IC}$

Minimal revision followed by abduction

Modeling the Firing Squad Example

- $\mathcal{P}$

$$
\begin{array}{lcl}
s & \leftarrow & e \\
ra & \leftarrow & s \\
rb & \leftarrow & s \\
d & \leftarrow & ra \\
d & \leftarrow & rb \\
a & \leftarrow & \neg d
\end{array}
$$

Modeling the Firing Squad Example

- $\mathcal{P}$

$$
\begin{array}{lcl}
s & \leftarrow & e \\
ra & \leftarrow & s \\
rb & \leftarrow & s \\
d & \leftarrow & ra \\
d & \leftarrow & rb \\
a & \leftarrow & \neg d
\end{array}
$$

- $\mathcal{M}_{\mathcal{P}}$

$$\langle \emptyset, \emptyset \rangle$$

Modeling the Firing Squad Example

- $\mathcal{P}$

$$\begin{array}{lcl} s & \leftarrow & e \\ ra & \leftarrow & s \\ rb & \leftarrow & s \\ d & \leftarrow & ra \\ d & \leftarrow & rb \\ a & \leftarrow & \neg d \end{array}$$

- $\mathcal{M}_{\mathcal{P}}$

$$\langle \emptyset, \emptyset \rangle$$

- $\mathcal{A}_{\mathcal{P}}$

$$\{e \leftarrow \top, e \leftarrow \bot\}$$

Modeling the Firing Squad Example

- $\mathcal{P}$

$$\begin{array}{lcl} s & \leftarrow & e \\ ra & \leftarrow & s \\ rb & \leftarrow & s \\ d & \leftarrow & ra \\ d & \leftarrow & rb \\ a & \leftarrow & \neg d \end{array}$$

- $\mathcal{M}_{\mathcal{P}}$

$$\langle \emptyset, \emptyset \rangle$$

- $\mathcal{A}_{\mathcal{P}}$

$$\{e \leftarrow \top, e \leftarrow \bot\}$$

- **Observations**
  - $\mathcal{E}_{\top} = \{e \leftarrow \top\}$ **explains** $\{s, ra, rb, d, \neg a\}$

Modeling the Firing Squad Example

- $\mathcal{P}$

$$\begin{array}{lcl} s & \leftarrow & e \\ ra & \leftarrow & s \\ rb & \leftarrow & s \\ d & \leftarrow & ra \\ d & \leftarrow & rb \\ a & \leftarrow & \neg d \end{array}$$

- $\mathcal{M}_{\mathcal{P}}$

$$\langle \emptyset, \emptyset \rangle$$

- $\mathcal{A}_{\mathcal{P}}$

$$\{e \leftarrow \top, e \leftarrow \bot\}$$

- **Observations**
  - $\mathcal{E}_{\top} = \{e \leftarrow \top\}$ **explains** $\{s, ra, rb, d, \neg a\}$
  - $\mathcal{E}_{\bot} = \{e \leftarrow \bot\}$ **explains** $\{\neg s, \neg ra, \neg rb, \neg d, a\}$

Modeling the Firing Squad Example

- $\mathcal{P}$

$$
\begin{array}{lcl}
s & \leftarrow & e \\
ra & \leftarrow & s \\
rb & \leftarrow & s \\
d & \leftarrow & ra \\
d & \leftarrow & rb \\
a & \leftarrow & \neg d
\end{array}
$$

- $\mathcal{M}_{\mathcal{P}}$

$$\langle \emptyset, \emptyset \rangle$$

- $\mathcal{A}_{\mathcal{P}}$

$$\{e \leftarrow \top, e \leftarrow \bot\}$$

- **Observations**
  - $\mathcal{E}_{\top} = \{e \leftarrow \top\}$ **explains** $\{s, ra, rb, d, \neg a\}$
  - $\mathcal{E}_{\bot} = \{e \leftarrow \bot\}$ **explains** $\{\neg s, \neg ra, \neg rb, \neg d, a\}$
  - $\{\neg s, ra\}$ **cannot be explained**

The Firing Squad Examples

- Observations
  - $\mathcal{E}_\top = \{e \leftarrow \top\}$ explains $\{s, ra, rb, d, \neg a\}$
  - $\mathcal{E}_\bot = \{e \leftarrow \bot\}$ explains $\{\neg s, \neg ra, \neg rb, \neg d, a\}$
  - $\{\neg s, ra\}$ **cannot be explained**
- **If the prisoner is alive, then the captain did not signal**

$$cond(a, \neg s) : \mathcal{P} \Rightarrow \mathcal{P} \cup \mathcal{E}_\bot \Rightarrow true$$

The Firing Squad Examples

- Observations
  - $\mathcal{E}_\top = \{e \leftarrow \top\}$ explains $\{s, ra, rb, d, \neg a\}$
  - $\mathcal{E}_\bot = \{e \leftarrow \bot\}$ explains $\{\neg s, \neg ra, \neg rb, \neg d, a\}$
  - $\{\neg s, ra\}$ cannot be explained
- **If the prisoner is alive, then the captain did not signal**

$$cond(a, \neg s) : \mathcal{P} \Rightarrow \mathcal{P} \cup \mathcal{E}_\bot \Rightarrow true$$

- **If rifleman A shot, then rifleman B shot as well**

$$cond(ra, rb) : \mathcal{P} \Rightarrow \mathcal{P} \cup \mathcal{E}_\top \Rightarrow true$$

The Firing Squad Examples

- Observations
  - $\mathcal{E}_\top = \{e \leftarrow \top\}$ explains $\{s, ra, rb, d, \neg a\}$
  - $\mathcal{E}_\bot = \{e \leftarrow \bot\}$ explains $\{\neg s, \neg ra, \neg rb, \neg d, a\}$
  - $\{\neg s, ra\}$ **cannot be explained**
- **If the prisoner is alive, then the captain did not signal**

$$\mathit{cond}(a, \neg s) : \mathcal{P} \Rightarrow \mathcal{P} \cup \mathcal{E}_\bot \Rightarrow \mathit{true}$$

- **If rifleman A shot, then rifleman B shot as well**

$$\mathit{cond}(ra, rb) : \mathcal{P} \Rightarrow \mathcal{P} \cup \mathcal{E}_\top \Rightarrow \mathit{true}$$

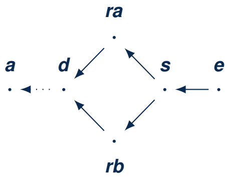
- **If the captain gave no signal and rifleman A decides to shoot, then the court did not order an execution**

$$\mathit{cond}(\{\neg s, ra\}, \neg e) : \mathcal{P} \Rightarrow \mathit{rev}(\mathcal{P}, ra) \cup \mathcal{E}_\bot \Rightarrow \mathit{true}$$

The Last Firing Squad Example Revisited

- If the captain gave no signal and rifleman A decides to shoot, then the court did not order an execution

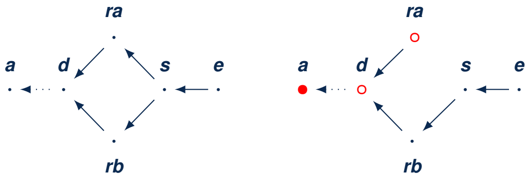
$$\mathcal{P} \Rightarrow \textit{rev}(\mathcal{P}, \textit{ra}) \cup \mathcal{E}_\bot$$



The Last Firing Squad Example Revisited

- If the captain gave no signal and rifleman A decides to shoot, then the court did not order an execution

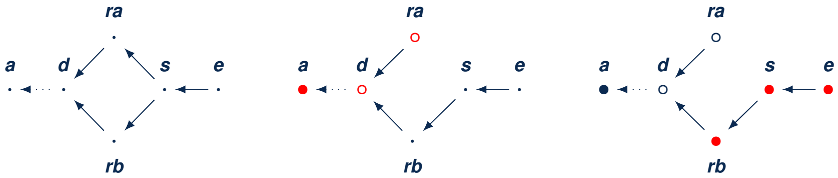
$$\mathcal{P} \Rightarrow \boldsymbol{rev}(\mathcal{P}, \boldsymbol{ra}) \cup \mathcal{E}_\perp$$



The Last Firing Squad Example Revisited

- If the captain gave no signal and rifleman A decides to shoot, then the court did not order an execution

$$\mathcal{P} \Rightarrow \textit{rev}(\mathcal{P}, \textit{ra}) \cup \mathcal{E}_\perp$$



An Alternative Approach

- Katrin Schulz: Minimal Models vs. Logic Programming: The Case of Counterfactual Conditionals. Journal of Applied Non-Classical Logics, 24, 153-168: 2014
- **Lemma** For all $n \in \mathbb{N}$, we find $\Phi_{rev(\mathcal{P},\mathcal{S})} \uparrow n \subseteq \Psi_{\mathcal{P},\mathcal{S}} \uparrow n \subseteq \Phi_{rev(\mathcal{P},\mathcal{S})} \uparrow (n+1)$
- **Theorem** lfp $\Phi_{rev(\mathcal{P},\mathcal{S})} =$ lfp $\Psi_{\mathcal{P},\mathcal{S}}$

Subjunctive Conditionals–The Forest Fire Example

- **If there had not been so many dry leaves on the forest floor, then the forest fire would not have occurred** Byrne 2007

$$\begin{array}{lcl} \mathcal{P} & = & \{ff \leftarrow l \wedge \neg ab,\; l \leftarrow \top,\; ab \leftarrow \neg dl,\; dl \leftarrow \top\} \\ \mathcal{M}_{\mathcal{P}} & = & \langle \{dl, l, ff\}, \{ab\} \rangle \end{array}$$

Subjunctive Conditionals–The Forest Fire Example

- **If there had not been so many dry leaves on the forest floor, then the forest fire would not have occurred** Byrne 2007

$$\mathcal{P} = \{ff \leftarrow l \wedge \neg ab,\ l \leftarrow \top,\ ab \leftarrow \neg dl,\ dl \leftarrow \top\}$$
$$\mathcal{M}_{\mathcal{P}} = \langle\{dl, l, ff\}, \{ab\}\rangle$$

- **Subjunctive conditional** $cond(\neg dl, \neg ff)$

Subjunctive Conditionals–The Forest Fire Example

- If there had not been so many dry leaves on the forest floor, then the forest fire would not have occurred — Byrne 2007

$$\mathcal{P} = \{ff \leftarrow l \wedge \neg ab,\ l \leftarrow \top,\ ab \leftarrow \neg dl,\ dl \leftarrow \top\}$$
$$\mathcal{M}_{\mathcal{P}} = \langle\{dl, l, ff\}, \{ab\}\rangle$$

- Subjunctive conditional $cond(\neg dl, \neg ff)$

$$rev(\mathcal{P}, \neg dl) = \{ff \leftarrow l \wedge \neg ab,\ l \leftarrow \top,\ ab \leftarrow \neg dl,\ dl \leftarrow \bot\}$$

Subjunctive Conditionals–The Forest Fire Example

- If there had not been so many dry leaves on the forest floor, then the forest fire would not have occurred — Byrne 2007

$$\mathcal{P} = \{ff \leftarrow l \wedge \neg ab,\ l \leftarrow \top,\ ab \leftarrow \neg dl,\ dl \leftarrow \top\}$$
$$\mathcal{M}_{\mathcal{P}} = \langle \{dl, l, ff\}, \{ab\} \rangle$$

- Subjunctive conditional $cond(\neg dl, \neg ff)$

$$rev(\mathcal{P}, \neg dl) = \{ff \leftarrow l \wedge \neg ab,\ l \leftarrow \top,\ ab \leftarrow \neg dl,\ dl \leftarrow \bot\}$$

	$\Phi_{rev(\mathcal{P},\neg dl)}$
$\uparrow 0$	$\langle \emptyset, \emptyset \rangle$
$\uparrow 1$	$\langle \{l\}, \{dl\} \rangle$
$\uparrow 2$	$\langle \{l, ab\}, \{dl\} \rangle$
$\uparrow 3$	$\langle \{l, ab\}, \{dl, ff\} \rangle$

The Extended Forest Fire Example

- **If there had not been so many dry leaves on the forest floor, then the forest fire would not have occurred** Pereira, Dietz, H. 2014

$$
\begin{aligned}
\mathcal{P} &= \{ff \leftarrow l \wedge \neg ab_1,\ ff \leftarrow a \wedge \neg ab_2, \\
&\quad l \leftarrow \top,\ ab_1 \leftarrow \neg dl,\ dl \leftarrow \top,\ ab_2 \leftarrow \bot\} \\
\mathcal{M}_{\mathcal{P}} &= \langle \{dl, l, ff\}, \{ab_1, ab_2\} \rangle \\
rev(\mathcal{P}, \neg dl) &= \{ff \leftarrow l \wedge \neg ab_1,\ ff \leftarrow a \wedge \neg ab_2, \\
&\quad l \leftarrow \top,\ ab_1 \leftarrow \neg dl,\ dl \leftarrow \bot,\ ab_2 \leftarrow \bot\} \\
\mathcal{M}_{rev(\mathcal{P}, \neg dl)} &= \langle \{l, ab_1\}, \{dl, ab_2\} \rangle
\end{aligned}
$$

Future Work

- Evaluating subjunctive conditionals
- Applying abduction before evaluating the consequent of a conditional
- Experiments