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Reasoning with Forest Logic Programs Using Fully Enriched Automata

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Introduction

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Forest Logic Programs:

- decidable fragment of Open Answer Set Programming
- non-monotonic language and rule-based syntax
- open domain semantics
- can simulate reasoning with the expressive DL $\mathcal{SHOQ}$

Previous work:

- non-deterministic tableau algorithms: 2NEXPTIME, NEXPTIME running time
- exact complexity characterization still open

Current work:

- encoding of reasoning with FoLPs into emptiness checking of fully enriched automata $\Longrightarrow$ EXPTIME procedure $\Longrightarrow$ worst-case optimal

Closed vs. Open World Reasoning

$$\begin{aligned} fail(X) &\leftarrow not\ pass(X) \\ pass(john) &\leftarrow \end{aligned}$$

$\rightarrow$ *ground* the program with all constants ($john$):

$$\begin{aligned} fail(john) &\leftarrow not\ pass(john) \\ pass(john) &\leftarrow \end{aligned}$$

$\rightarrow$ answer set: $\{pass(john)\}$.
$\rightarrow$ *fail* is not satisfiable:

- assume the presence of anonymous objects – *open domains*
- e.g. with universe $\{john, x\}$, *fail* becomes satisfiable

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Open Answer Set Programming

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Enhancing Answer Set Programming with open domains:

Syntax

same as the syntax of *function-free* Answer Set Programming

Semantics (OASP)

- (U, M) is an *open answer set* of an OASP (FoLP) P, iff $U \supseteq cts(P)$ and M is an answer set of P_U

When $U = \{john, x\}$, P_U:

$$\begin{aligned} fail(john) &\leftarrow not\ pass(john) \\ fail(x) &\leftarrow not\ pass(x) \\ pass(john) &\leftarrow \end{aligned}$$

$M = \{pass(john), fail(x)\}$ is an answer set of P_U: $\rightsquigarrow$
$(\{john, x\}, \{pass(john), fail(x)\})$ is an open answer set!

Forest Logic Programs

OASP is undecidable: syntactical restrictions to achieve decidability;

Forest Logic Programs

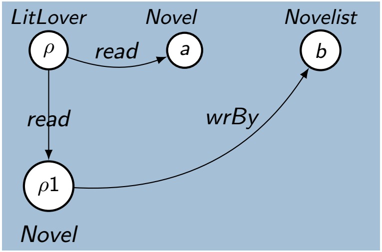
- allow only for unary and binary predicates
- tree-shaped rules: *forest model property*
- a special type of unsafe rules: *free rules*
- facts

$$
\begin{aligned}
r_1 &: & LitLover(X) &\leftarrow read(X, Y_1), read(X, Y_2), \\
& & & \quad Novel(Y_1), Novel(Y_2), Y_1 \neq Y_2 \\
r_2 &: & Novel(X) &\leftarrow wrBy(X, Y), Novelist(Y) \\
r_3 &: & Novelist(X) &\leftarrow wrote(X, Y), Novel(Y) \\
r_4 &: & read(X, Y) \vee not\ read(X, Y) &\leftarrow \\
r_5 &: & wrBy(X, Y) \vee not\ wrBy(X, Y) &\leftarrow \\
r_6 &: & wrote(X, Y) \vee not\ wrote(X, Y) &\leftarrow \\
f_1 &: & Novel(a) &\leftarrow \\
f_2 &: & Novelist(b) &\leftarrow
\end{aligned}
$$

Forest model property

A unary predicate is satisfiable iff it is satisfied by a forest-shaped model

$r_1 : LitLover(X) \leftarrow read(X, Y_1), read(X, Y_2),$
$\qquad Novel(Y_1), Novel(Y_2), Y_1 \neq Y_2.$
$r_2 : Novel(X) \leftarrow wrBy(X, Y), Novelist(Y).$
$r_3 : Novelist(X) \leftarrow wrote(X, Y), Novel(Y).$
$r_4 : read(X, Y) \vee not\ read(X, Y) \leftarrow .$
$r_5 : wrBy(X, Y) \vee not\ wrBy(X, Y) \leftarrow .$
$r_6 : wrote(X, Y) \vee not\ wrote(X, Y) \leftarrow .$
$f_1 : Novel(a).$
$f_2 : Novelist(b).$



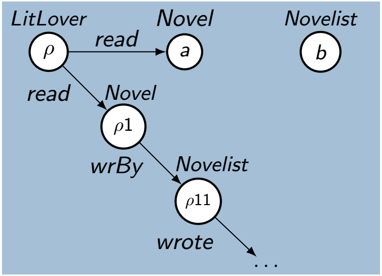
(U, M) with:

- $U = \{\rho, \rho 1, a, b\}$, and
- $M = \{LitLover(\rho), Novel(a), read(\rho, \rho 1), \ldots\}$

is a forest model which satisfies *LitLover*

Forest Models are Well-Supported

$r_1 : LitLover(X) \leftarrow read(X, Y_1), read(X, Y_2), Novel(Y_1), Novel(Y_2), Y_1 \neq Y_2.$
$r_2 : Novel(X) \leftarrow wrBy(X, Y), Novelist(Y).$
$r_3 : Novelist(X) \leftarrow wrote(X, Y), Novel(Y).$
$r_4 : read(X, Y) \vee not\ read(X, Y) \leftarrow .$
$r_5 : wrBy(X, Y) \vee not\ wrBy(X, Y) \leftarrow .$
$r_6 : wrote(X, Y) \vee not\ wrote(X, Y) \leftarrow .$
$f_1 : Novel(a).$
$f_2 : Novelist(b).$



(U, M) with:

- $U = \{\rho, \rho 1, \ldots, a, b\}$, and
- $M = \{LitLover(\rho), Novel(a), Novelist(b)\ read(\rho, \rho 1), Novel(\rho 1), \ldots\}$

is not a forest model!

Constructing well-supported models

Done in the past using tableaux algorithms:

- blocking mechanism incorporates a well-supportedness check
- usually non-deterministic: 2NExpTime, NExpTime running times
- worst-case optimal (ExpTime) AND/OR tableaux algorithm devised for the case of CoLPs (FoLPs$\setminus$constants)
- AND/OR technique does not generalize to FoLPs
- complexity gap: satisfiability checking w.r.t. FoLPs was known to be ExpTime-hard

Fully Enriched Automata

- Run on labeled forests
- Introduced as a device to reason with hybrid graded μ-calculus

$A = \langle \Sigma, b, Q, \delta, q_0, \mathcal{F} \rangle$:

- Σ is a finite input alphabet
- $b > 0$ is a counting bound
- Q is a finite set of states
- $\delta : Q \times \Sigma \to B^+(D_b \times Q)$ - the transition function, where:
  - $B^+(Y)$ is the set of positive Boolean formulas over Y
  - $D_b = \{\langle 0\rangle, \langle 1\rangle, \ldots, \langle b\rangle\} \cup \{[0], [1], \ldots, [b]\} \cup \{-1, \varepsilon, \langle root\rangle, [root]\}$
- $q_0 \in Q$ - the initial state
- $\mathcal{F} = \{\mathcal{F}_1, \mathcal{F}_2, \ldots, \mathcal{F}_k\}$, where $\mathcal{F}_1 \subseteq \mathcal{F}_2 \subseteq \ldots \subseteq \mathcal{F}_k = Q$ is a *parity acceptance condition*

Emptiness checking for a FEA A as above with n states can be decided in time $(b+2)^{\mathcal{O}(n^3 \cdot k^2 \cdot \log k \cdot \log b^2)}$.

Reasoning with FoLPs Using FEA

For every FoLP P and unary predicate p construct a class of FEA $A^{p,P}_{\rho,\theta}$:

- ρ is a designated constant or anonymous node
- θ fixes a label for each root node of accepted forests
- states of the form $q_{t,a}$, q_{t,r_a}, etc. where t is a term pattern (a designated constant or *), a is a unary predicate, r_a is a unary rule, etc.
- number of states: polynomial in the size of P
- parity acceptance condition: $\mathcal{F} = (\mathcal{F}_1, \mathcal{F}_2)$
  - $\mathcal{F}_1 = \{q_{t,a}, q_{t_1,t_2,f} \mid a/f$ a unary/binary predicate; t, t_1 and t_2 term patterns $\}$,
  - $\mathcal{F}_2 = Q$
  - exploited for checking well-supportedness

For details about the encoding, please check the paper!

Properties of the Encoding

For a FoLP P and a unary predicate symbol p, p is satisfiable w.r.t. P iff there exists an automaton $A^{p,P}_{\rho,\theta}$ whose language is non-empty.

Satisfiability checking of unary predicates with respect to FoLPs is EXPTIME-complete.

f-hybrid KBs: pairs (Σ, P)

- Σ a $\mathcal{SHOQ}$ kb, P a FoLP: no restriction on signature sharing
- a unary predicate p is satisfiable w.r.t. (Σ, P) iff it is satisfiable w.r.t. $\Theta(\Sigma) \cup P$, where Θ is a polynomial and modular translation from $\mathcal{SHOQ}$ to FoLPs.

Satisfiability checking of unary predicates with respect to f-hybrid KBs is EXPTIME-complete.

Conclusions

The result closes an open problem: exact complexity characterization of FoLPs

FEAs - elegant device for encoding

- accept forests as input
- parity acceptance condition to check well-supportedness
- additional addressing and term matching mechanisms needed

Existing work on AND/OR tableau reasoners for CoLPs (FoLPs minus constants):

- how can it be lifted to FoLPs?

Questions?