ASPMT(QS): Non-Monotonic Spatial Reasoning with Answer Set Programming Modulo Theories

Przemysław Andrzej Wałęga[1], Mehul Bhatt[2], and Carl Schultz[3]

[1] Institute of Philosophy, University of Warsaw, Poland,

[2] University of Bremen, Germany and
The DesignSpace Group

[3] University of Munster, Germany.

Lexington, USA, 30 September 2015

Motivations

'Space, with its manifold layers of structure, has been an inexhaustible source of intellectual fascination since Antiquity. [. . .] In this long intellectual history, however, one relatively recent, yet crucial, event stands out: the rise of the logical stance in geometry.' [1]

[1] Handbook of spatial logics

Motivations

'Space, with its manifold layers of structure, has been an inexhaustible source of intellectual fascination since Antiquity. [...] In this long intellectual history, however, one relatively recent, yet crucial, event stands out: the rise of the logical stance in geometry.' [1]

However, no systems currently exist that are capable of efficient and general nonmonotonic spatial reasoning, which is a key requirement in systems that aim to model a wide range of dynamic application domains.

[1] Handbook of spatial logics

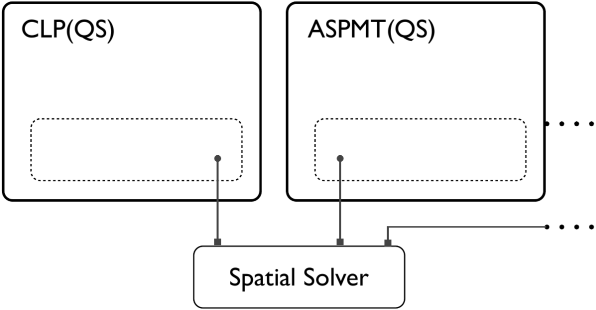
ASPMT(QS)

ASPMT(QS) = ASP Modulo Theories with Qualitative Space

ASPMT(QS) = ASP Modulo Theories with Qualitative Space

ASPMT(QS) contains:

1. Spatial solver – our spatial reasoning module,
2. ASPMT2SMT (by M. Bartholomew and J. Lee) – a compiler translating a tight fragment of ASPMT into SMT instances,
3. Z3 – the SMT solver (for arithmetic over reals).



Domain Entities in $\mathcal{QS}$

Domain entities in $\mathcal{QS}$ are *circles, triangles, points*, *segments*, etc.:

- a *point* is a pair of reals x, y,
- a *line segment* is a pair of end points p_1, p_2, $(p_1 \neq p_2)$,
- a *circle* is a centre point p and a real radius r $(0 < r)$,
- a *triangle* is a triple of vertices (points) p_1, p_2, p_3 such that p_3 is to the *left of* segment p_1, p_2.

A range of spatial relations with the corresponding polynomial encodings, e.g.,

- *Relative Orientation.* *Left, right, collinear* (between *point* and *segment*), *parallel, perpendicular* (between *segments*),
- *Mereotopology.* *Part-whole* and *contact* relations between regions.

A *parametric function* is an n–ary function:

$$f_n : D_1 \times D_2 \times \cdots \times D_n \to \mathbb{R},$$

where D_i is a type of a spatial object, e.g., $Points$, $Circles$, etc.

Example:

x-coordinate $\quad x : Circles \to \mathbb{R}$
y-coordinate $\quad y : Circles \to \mathbb{R}$
radius $\quad r : Circles \to \mathbb{R}$

Then $c \in Cirlces$ may be described as:
$x(c) = 1.23 \;\wedge\; y(c) = -0.13 \;\wedge\; r(c) = 2.$

A *qualitative spatial relation* is an n-ary predicate:

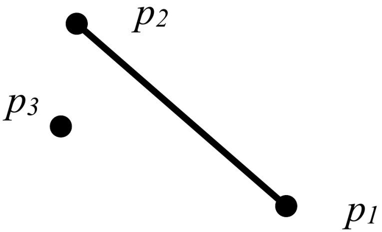
$$Q_n \subseteq D_1 \times \cdots \times D_n,$$

where D_i is a type of a spatial object.

Example:

two circles are equal $\quad rccEQ \subseteq Circles \times Circles$

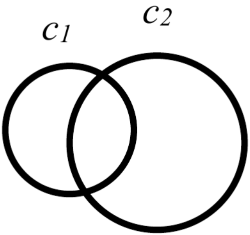
Polynomial encoding of a relation "the point is to the left of the line":

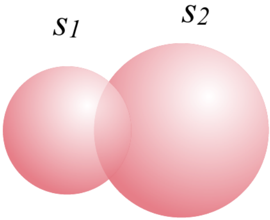


$$left(p_3, p_1p_2) \leftrightarrow x_2y_3 + x_1y_2 + x_3y_1 - y_2x_3 - y_1x_2 - y_3x_1 > 0$$

Analytic spatial reasoning

Polynomial encoding of a relation "two circles partially overlap":





$$po(c1, c2) \leftrightarrow d^2 = (x_1 - x_2)^2 + (y_1 - y_2)^2$$
$$\wedge\, d^2 < (r_1 + r_2)^2 \wedge d^2 > (r_1 - r_2)^2$$

Proposition

Each relation of RCC–5 in the domain of circles and convex polygons with finite number of vertices may be defined in ASPMT(QS).

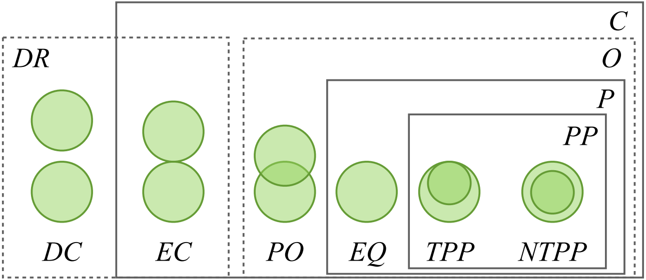


Figure : RCC relations.

Example – encodings of RCC-5 relations

```
%----------RCC5 circle,circle
%rcc eq
rccEQ(C1,C2)=true <- (x(C1)=X1 & y(C1)=Y1 & r(C1)=R1 & x(C2)=X2 & y(C2)=Y2 & r(C2)=R2) &
(X1=X2 & Y1=Y2 & R1=R2).
rccEQ(C1,C2)=false <- (x(C1)=X1 & y(C1)=Y1 & r(C1)=R1 & x(C2)=X2 & y(C2)=Y2 & r(C2)=R2) &
not (X1=X2 & Y1=Y2 & R1=R2).

%rcc dr
rccDR(C1,C2)=true <- (x(C1)=X1 & y(C1)=Y1 & r(C1)=R1 & x(C2)=X2 & y(C2)=Y2 & r(C2)=R2) &
(X1-X2)*(X1-X2)+(Y1-Y2)*(Y1-Y2) >= (R1+R2)*(R1+R2).
rccDR(C1,C2)=false <- (x(C1)=X1 & y(C1)=Y1 & r(C1)=R1 & x(C2)=X2 & y(C2)=Y2 & r(C2)=R2) &
not (X1-X2)*(X1-X2)+(Y1-Y2)*(Y1-Y2) >= (R1+R2)*(R1+R2).

%rcc pp
rccPP(C1,C2)=true <- (x(C1)=X1 & y(C1)=Y1 & r(C1)=R1 & x(C2)=X2 & y(C2)=Y2 & r(C2)=R2) &
( R1<R2 & (X1-X2)*(X1-X2)+(Y1-Y2)*(Y1-Y2) <= (R1-R2)*(R1-R2)).
rccPP(C1,C2)=false <- (x(C1)=X1 & y(C1)=Y1 & r(C1)=R1 & x(C2)=X2 & y(C2)=Y2 & r(C2)=R2) &
not ( R1<R2 & (X1-X2)*(X1-X2)+(Y1-Y2)*(Y1-Y2) <= (R1-R2)*(R1-R2) ).

%rcc ppi
rccPPi(C2,C1)=B <- rccPP(C1,C2)=B.

%rcc po
rccPO(C1,C2)=true <- (x(C1)=X1 & y(C1)=Y1 & r(C1)=R1 & x(C2)=X2 & y(C2)=Y2 & r(C2)=R2) &
( (X1-X2)*(X1-X2)+(Y1-Y2)*(Y1-Y2) > (R1-R2)*(R1-R2) & (X1-X2)*(X1-X2)+(Y1-Y2)*(Y1-Y2)
< (R1+R2)*(R1+R2)).
rccPO(C1,C2)=false <- (x(C1)=X1 & y(C1)=Y1 & r(C1)=R1 & x(C2)=X2 & y(C2)=Y2 & r(C2)=R2) &
not ( (X1-X2)*(X1-X2)+(Y1-Y2)*(Y1-Y2) > (R1-R2)*(R1-R2) & (X1-X2)*(X1-X2)+(Y1-Y2)*(Y1-Y2)
< (R1+R2)*(R1+R2)).
```

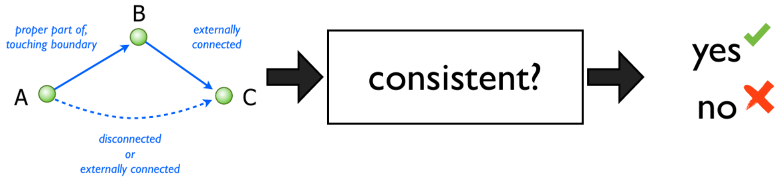
Relations from Qualitative Calculi

Proposition

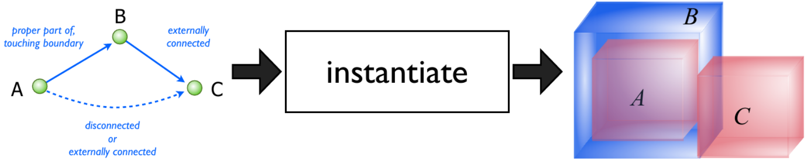
Each relation of Interval Algebra (IA) and Rectangle Algebra (RA) may be defined in ASPMT(QS).

Proposition

Each relation of Cardinal Direction Calculus (CDC) may be defined in ASPMT(QS).



B
proper part of, touching boundary
externally connected
A
C
disconnected or externally connected
consistent?
yes
no



B
proper part of, touching boundary
externally connected
A
C
disconnected or externally connected
instantiate
B
A
C

Input program

ASPMT(QS) supports:

- connectives: `&`, `|`, `not`, `->`, `<-`,
- arithmetic operators: <, <=, >=, >, =, !=, +, =, *.

Additionally, native entities:

- `sorts` for geometric objects types, e.g., `point`, `segment`, `circle`, `triangle`,
- parametric functions describing objects parameters e.g., `x(point)`, `r(circle)`,
- qualitative relations, e.g., `rccEC(circle, circle)`, `coincident(point, circle)`.

The input program is divided into declarations of:

- `sorts` (data types);
- `objects` (particular elements of given types);
- `constants` (functions);
- `variables` (variables associated with declared types).

The second part of the program consists of rules.

Output:
ASPMT(QS) checks if there exists a stable model.

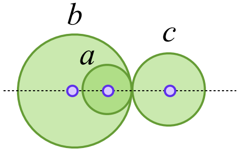
In the case of a positive answer, a parametric model is shown.

Example: combining topology and relative orientation

Given three circles a, b, c let a be proper part of b, b discrete from c, and a in contact with c, declared as follows:

```
:- sorts
   circle.
:- objects
   a, b, c       ::  circle.

rccPP(a,b)=true.
rccDR(b,c)=true.
rccC(a,c)=true.
```



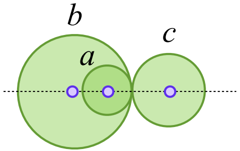
Example: combining topology and relative orientation

ASPMT(QS) infers that:

- a is a *tangential proper part* of b,
- both a and b are *externally connected* to c.

The output is:

```
r(a) = 1.0          r(b) = 2.0          r(c) = 1.0
x(a) = 1.0          x(b) = 0.0          x(c) = 3.0
y(a) = 0.0          y(b) = 0.0          y(c) = 0.0
rccTPP(a,b) = true
rccEC(a,c) = true
rccEC(b,c) = true
```



Example: combining topology and relative orientation

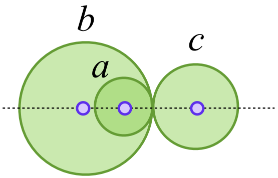
We then add an additional constraint that the centre of a is *left of* the segment between the centres b to c:

```
...
left_of(center(a),center(b),center(c)).
```

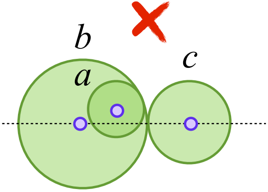
The output is:

```
UNSATISFIABLE;
```

Meaning that (b) is inconsistent, i.e., the centres must be *collinear* (a).

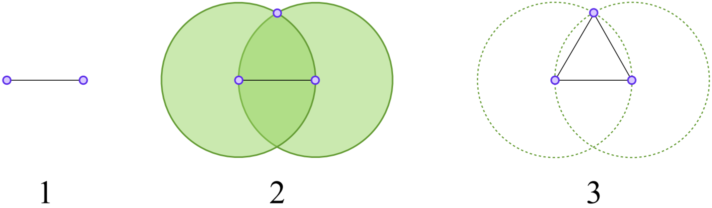


(a)



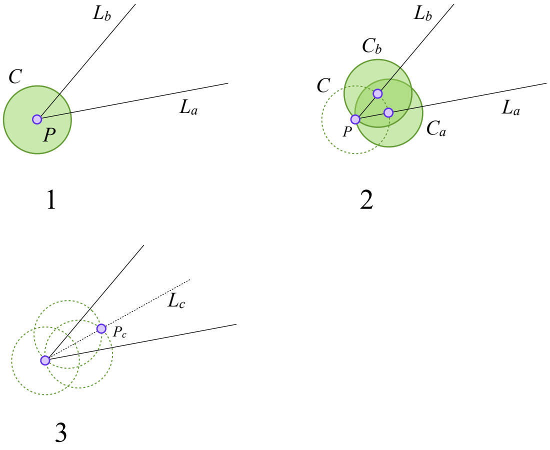
(b)

Euclid construction: equilateral triangle



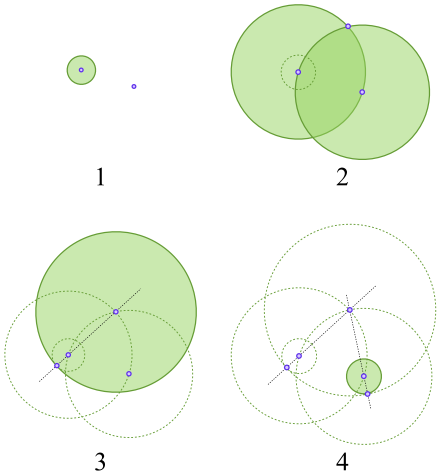
ASPMT(QS) infers that the constructed triangle is equilateral.

Euclid construction: bisecting angle



ASPMT(QS) infers that the constructed ray bisects the angle.

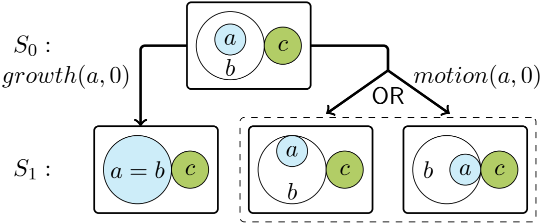
Euclid construction: compass equivalence theorem



ASPMT(QS) infers that the constructed circle has a same radius as the initial circle.

Example: indirect effects

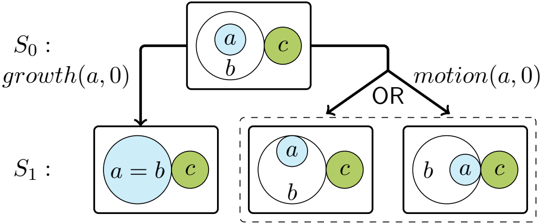
Initially three cells a, b, c; a is a non-tangential proper part of b and b is externally connected to c.



1) a grows in step S_0. In S_1 a is equal to b. The inferred indirect effect: a is externally connected to c in S_1.

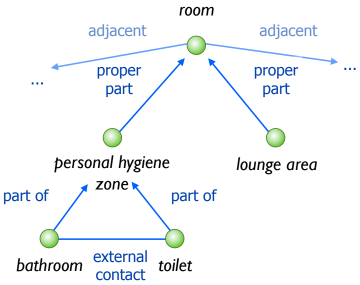
Example: indirect effects

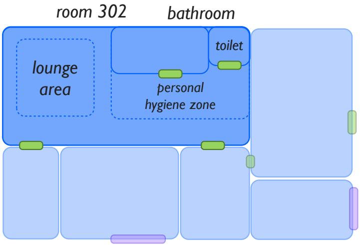
Initially three cells a, b, c; a is a non-tangential proper part of b and b is externally connected to c.



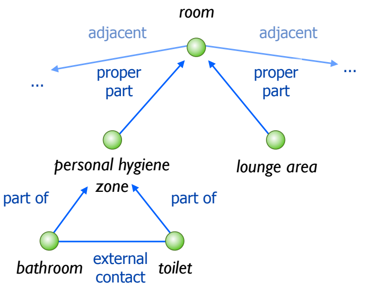
2) a moves in step S_0. In S_1 a is tangential proper part of b. The inferred indirect effect: a is externally connected to c or disconnected from c in S_1.

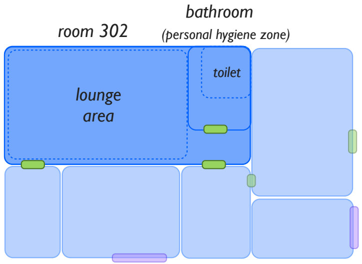
Left: qualitative spatial graph of the architectural building:





Right: nitial floor plan consistent with qualitative specification:





Additional requirements for dimensions of:

- room has $20m^2$,
- lounge area has $15m^2$,
- bathroom has $4m^2$,
- toilet has $3m^2$.

ASPMT(QS) infers that the design has to be changed at a qualitative level. Toilet needs to be located inside the bathroom.

Evaluation

Table : *Cumulative results of performed tests. "—" indicates that the problem can not be formalised, "^{I}" indicates that indirect effects can not be formalised, "^{D}" indicates that default rules can not be formalised.*

Problem	Clingo	GQR	CLP($\mathcal{QS}$)	ASPMT(QS)
Growth	0.004s^{I}	0.014s^{I,D}	1.623s^{D}	0.396s
Motion	0.004s^{I}	0.013s^{I,D}	0.449s^{D}	15.386s
Attach I	0.008s^{I}	—	3.139s^{D}	0.395s
Attach II	—	—	2.789s^{D}	0.642s

Summarize

ASPMT(QS):

- is a novel approach for reasoning about spatial change within a KR paradigm,
- can model behaviour patterns that characterise commonsense reasoning about space, actions, and change,
- is capable of combining qualitative and quantitative spatial information when reasoning non-monotonically.

Future work:

- publish a downloadable version of ASPMT(QS) (soon).
- extend ASPMT(QS) system to perform more complex spatio-temporal reasoning, and apply to practical problems: computer-aided architecture design, mobile robots control, etc.

Thank you

p.a.walega@gmail.com

This research is supported by:

(a) the Polish National Science Centre grant 2011/02/A/HS1/0039; and

(b) the DesignSpace Research Group www.design-space.org (University of Bremen)