Characterising and Explaining Inconsistency in Logic Programs

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LPNMR'15

Unsatisfiable

```
clingo version 4.6.0
Solving...
UNSATISFIABLE

Models       : 0
Calls        : 1
Time         : 0.013s (Solving: 0.00s 1st Model: 0.00s Unsat: 0.00s)
CPU Time     : 0.000s
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$\mathcal{P}$ **inconsistent**: only answer set $Lit_{\mathcal{P}}$ or no answer sets

Inconsistency

Answer Set (no NAF)

$\vdots$

$S = Lit_{\mathcal{P}}$ if S contains some a and $\neg a$

Inconsistency

Answer Set (no NAF)

$\vdots$

$S = Lit_{\mathcal{P}}$ if S contains some a and $\neg a$

Answer Set (with NAF)

$\vdots$

S is an answer set of $\mathcal{P}$ if $S = \mathcal{AS}(\mathcal{P}^S)$.

Inconsistency – Contributions

Inconsistency scenarios:

1. $S = Lit_{\mathcal{P}}$
2. no answer set

Inconsistency – Contributions

Inconsistency scenarios:

1. $S = Lit_P$
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$\Longrightarrow$ **4 scenarios**

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Inconsistency scenarios:

1. $S = Lit_P$
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$\Longrightarrow$ **4 scenarios**

Reasons for inconsistency:

1. explicit negation ($\neg$)
2. NAF (`not`)

Inconsistency – Contributions

Inconsistency scenarios:

1. ~~$S = Lit_P$~~
2. ~~no answer set~~

$\Longrightarrow$ **4 scenarios**

Reasons for inconsistency:

1. explicit negation ($\neg$)
2. NAF (`not`)

$+$ **explicit negation ($\neg$) + NAF (`not`)**

Background

Background – 3-valued Stable Models

True $>$ Undefined $>$ False

- head at least as true as weakest body literal
- "minimal"

Example

$a \leftarrow \texttt{not } b$ $\quad$ $d \leftarrow$ $\quad$ $e \leftarrow \texttt{not } f$

$b \leftarrow \texttt{not } c$ $\quad$ $f \leftarrow \texttt{not } e$

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- $\langle \{b, d, e\}, \{a, c, f\} \rangle \quad \mathcal{U} = \emptyset$

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- $\langle \{b, d, f\}, \{a, c, e\} \rangle \quad \mathcal{U} = \emptyset$
- $\langle \{b, d\}, \{a, c\} \rangle \quad \mathcal{U} = \{e, f\}$ **well-founded**

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- $\langle \{b, d, e\}, \{a, c, f\} \rangle \quad \mathcal{U} = \emptyset \quad$ **M-stable**
- $\langle \{b, d, f\}, \{a, c, e\} \rangle \quad \mathcal{U} = \emptyset \quad$ **M-stable**
- $\langle \{b, d\}, \{a, c\} \rangle \quad \mathcal{U} = \{e, f\} \quad$ **well-founded**

Background – Translated Logic Program $\mathcal{P}'$

Example

$\neg a \leftarrow$ $\qquad$ $a \leftarrow$

- no answer set, 3-valued stable model not defined

Background – Translated Logic Program $\mathcal{P}'$

Example

$a' \leftarrow$ $\quad$ $a \leftarrow$

- no answer set, 3-valued stable model not defined

Background – Translated Logic Program $\mathcal{P}'$

Example

$a' \leftarrow$ $\quad$ $a \leftarrow$

- no answer set, 3-valued stable model not defined
- answer set, 3-valued stable model: $\{a, a'\}$

Background – Translated Logic Program $\mathcal{P}'$

Example

$a' \leftarrow$ $\quad$ $a \leftarrow$

- no answer set, 3-valued stable model not defined
- answer set, 3-valued stable model: $\{a, a'\}$

3-valued stable models of $\mathcal{P}$ correspond to those 3-valued stable models of $\mathcal{P}'$ with no a and a'

Background – Translated Logic Program $\mathcal{P}'$

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- answer set, 3-valued stable model: $\{a, a'\}$

3-valued stable models of $\mathcal{P}$ correspond to those 3-valued stable models of $\mathcal{P}'$ with no a and a'

From now on we assume $\mathcal{P}$ is inconsistent

Inconsistency Scenarios – Part 1 –

Inconsistency Scenarios

3 scenarios by using the well-founded model

- no well-founded model
- well-founded model

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  3. no answer sets

Scenario 1 & 2 – no well-founded model

Example (Scenario 1)

$$p \leftarrow r, s \qquad r \leftarrow$$

$$\neg p \leftarrow \qquad s \leftarrow$$

Scenario 1 & 2 – no well-founded model

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- only answer set $Lit_{\mathcal{P}} = \{p, \neg p, r, \neg r, s, \neg s\}$

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Example (Scenario 2)

$$p \leftarrow r, s \qquad r \leftarrow$$

$$\neg p \leftarrow \texttt{not}\ q \qquad s \leftarrow$$

Scenario 1 & 2 – no well-founded model

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Example (Scenario 2)

$$p \leftarrow r, s \qquad r \leftarrow$$

$$s \leftarrow$$

- no well-founded model
- no answer set

Culprits

Scenario 1 – no well-founded model & answer set $Lit_{\mathcal{P}}$

Example (Scenario 1)

$$p \leftarrow r, s \qquad r \leftarrow$$

$$\neg p \leftarrow \qquad s \leftarrow$$

- no well-founded model
- only answer set $Lit_{\mathcal{P}}$

Scenario 1 – no well-founded model & answer set $Lit_{\mathcal{P}}$

Example (Scenario 1)

$$p \leftarrow r, s \qquad r \leftarrow$$

$$\neg p \leftarrow \qquad s \leftarrow$$

- culprit set: $\{p, \neg p\}$

Scenario 1 – no well-founded model & answer set $Lit_{\mathcal{P}}$

Example (Scenario 1)

$$p \leftarrow r, s \qquad r \leftarrow$$

$$p' \leftarrow \qquad s \leftarrow$$

- culprit set: $\{p, \neg p\}$

Scenario 1 – no well-founded model & answer set $Lit_{\mathcal{P}}$

Example (Scenario 1)

$$p \leftarrow r, s \qquad r \leftarrow$$
$$p' \leftarrow \qquad s \leftarrow$$

- culprit set: $\{p, \neg p\}$
- well-founded model of $\mathcal{P}'$: $\langle \{p, p', r, s\}, \emptyset \rangle$

Scenario 2 – no well-founded model & no answer set

Example (Scenario 2)

$p \leftarrow r, s$ $\qquad$ $r \leftarrow$

$\neg p \leftarrow \texttt{not}\ q$ $\qquad$ $s \leftarrow$

- no well-founded model
- no answer set

Scenario 2 – no well-founded model & no answer set

Example (Scenario 2)

$p \leftarrow r, s$ $\qquad$ $r \leftarrow$

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Scenario 2 – no well-founded model & no answer set

Example (Scenario 2)

$$p \leftarrow r, s \qquad r \leftarrow$$

$$p' \leftarrow \texttt{not}\ q \qquad s \leftarrow$$

- culprit set: $\{p, \neg p\}$
- well-founded model: $\langle \{p, p', s, r\}, \{q\} \rangle$

Inconsistency Scenarios

3 scenarios by using the well-founded model

- no well-founded model
  1. only answer set $Lit_{\mathcal{P}}$
  2. no answer sets
- well-founded model
  3. no answer sets

Inconsistency Scenarios

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Inconsistency Scenarios

3 scenarios by using the well-founded model

- no well-founded model
  1. only answer set $Lit_{\mathcal{P}}$
     $\Rightarrow$ **culprit set**: $\{a, \neg a\}$ both strictly derivable
  2. no answer sets
- well-founded model
  3. no answer sets

Inconsistency Scenarios

3 scenarios by using the well-founded model

- no well-founded model
  1. only answer set $Lit_{\mathcal{P}}$
     $\Rightarrow$ **culprit set**: $\{a, \neg a\}$ both strictly derivable
     $\Rightarrow$ **reason**: explicit negation
  2. no answer sets
- well-founded model
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     $\Rightarrow$ **culprit set**: $\{a, \neg a\}$ one defeasibly derivable
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Inconsistency Scenarios & Culprits – Part 2 –

Scenario 3 – well-founded model & no answer set

Example (Scenario 3)

$r \leftarrow \texttt{not}\ s$ $\quad$ $q \leftarrow \texttt{not}\ s$ $\quad$ $p \leftarrow \texttt{not}\ r$

$s \leftarrow \texttt{not}\ r$ $\quad$ $\neg q \leftarrow \texttt{not}\ s$ $\quad$ $\neg p \leftarrow \texttt{not}\ r$

Scenario 3 – well-founded model & no answer set

Example (Scenario 3)

$r \leftarrow \texttt{not } s$ $\quad$ $q \leftarrow \texttt{not } s$ $\quad$ $p \leftarrow \texttt{not } r$

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- well-founded model: $\langle \emptyset, \emptyset \rangle$
- no answer set

Scenario 3 – well-founded model & no answer set

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$r \leftarrow \texttt{not}\ s$ $\quad$ $q \leftarrow \texttt{not}\ s$ $\quad$ $p \leftarrow \texttt{not}\ r$

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- no answer set

Example (Scenario 3)

$r \leftarrow \texttt{not}\ s$

$s \leftarrow \texttt{not}\ t$

$t \leftarrow \texttt{not}\ r$

$q \leftarrow \texttt{not}\ s$ $\quad$ $p \leftarrow \texttt{not}\ r$ $\quad$ $u \leftarrow \texttt{not}\ t$

$\neg q \leftarrow \texttt{not}\ s$ $\quad$ $\neg p \leftarrow \texttt{not}\ r$ $\quad$ $\neg u \leftarrow \texttt{not}\ t$

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$r \leftarrow \texttt{not}\ s$

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- well-founded model: $\langle \emptyset, \emptyset \rangle$
- no answer set

Inconsistency Scenarios

3 scenarios by using the well-founded model

- $\mathcal{P}$ has no well-founded model
  1. the only answer set is $Lit_{\mathcal{P}}$
  2. $\mathcal{P}$ has no answer sets
- $\mathcal{P}$ has a well-founded model
  3. $\mathcal{P}$ has no answer sets

Inconsistency Scenarios

3 scenarios by using the well-founded model

- $\mathcal{P}$ has no well-founded model
  1. the only answer set is $Lit_{\mathcal{P}}$
  2. $\mathcal{P}$ has no answer sets
- $\mathcal{P}$ has a well-founded model
  3. $\mathcal{P}$ has no answer sets
     - 3a $\mathcal{P}'$ has answer sets
     - 3b $\mathcal{P}'$ has no answer sets

Scenario 3a – well-founded model & no answer set

Example (Scenario 3a)

$r \leftarrow \texttt{not}\ s$ $\quad$ $q \leftarrow \texttt{not}\ s$ $\quad$ $p \leftarrow \texttt{not}\ r$

$s \leftarrow \texttt{not}\ r$ $\quad$ $\neg q \leftarrow \texttt{not}\ s$ $\quad$ $\neg p \leftarrow \texttt{not}\ r$

- well-founded model: $\langle \emptyset, \emptyset \rangle$
- no answer set

Scenario 3a – well-founded model & no answer set

Example (Scenario 3a)

$$r \leftarrow \texttt{not}\ s \qquad q \leftarrow \texttt{not}\ s \qquad p \leftarrow \texttt{not}\ r$$

$$s \leftarrow \texttt{not}\ r \qquad q' \leftarrow \texttt{not}\ s \qquad p' \leftarrow \texttt{not}\ r$$

Scenario 3a – well-founded model & no answer set

Example (Scenario 3a)

$r \leftarrow \texttt{not}\ s$ $\quad$ $q \leftarrow \texttt{not}\ s$ $\quad$ $p \leftarrow \texttt{not}\ r$

$s \leftarrow \texttt{not}\ r$ $\quad$ $q' \leftarrow \texttt{not}\ s$ $\quad$ $p' \leftarrow \texttt{not}\ r$

- answer sets: $S'_1 = \{p, p', s\}$

Scenario 3a – well-founded model & no answer set

Example (Scenario 3a)

$r \leftarrow \texttt{not}\ s$ $\quad$ $q \leftarrow \texttt{not}\ s$ $\quad$ $p \leftarrow \texttt{not}\ r$

$s \leftarrow \texttt{not}\ r$ $\quad$ $q' \leftarrow \texttt{not}\ s$ $\quad$ $p' \leftarrow \texttt{not}\ r$

- answer sets: $S'_1 = \{p, p', s\}$ and $S'_2 = \{q, q', r\}$

Scenario 3a – well-founded model & no answer set

Example (Scenario 3a)

$r \leftarrow \texttt{not}\ s$ $\quad$ $q \leftarrow \texttt{not}\ s$ $\quad$ $p \leftarrow \texttt{not}\ r$

$s \leftarrow \texttt{not}\ r$ $\quad$ $q' \leftarrow \texttt{not}\ s$ $\quad$ $p' \leftarrow \texttt{not}\ r$

- answer sets: $S'_1 = \{p, p', s\}$ and $S'_2 = \{q, q', r\}$
- culprit set: $\{p, \neg p, q, \neg q\}$

Scenario 3b – well-founded model & no answer set

Example (Scenario 3b)

$$r \leftarrow \texttt{not}\ s \qquad q \leftarrow \texttt{not}\ s \qquad p \leftarrow \texttt{not}\ r \qquad u \leftarrow \texttt{not}\ t$$

$$s \leftarrow \texttt{not}\ t \qquad \neg q \leftarrow \texttt{not}\ s \qquad \neg p \leftarrow \texttt{not}\ r \qquad \neg u \leftarrow \texttt{not}\ t$$

$$t \leftarrow \texttt{not}\ r$$

- well-founded model: $\langle \emptyset, \emptyset \rangle$
- no answer set

Scenario 3b – well-founded model & no answer set

Example (Scenario 3b)

$r \leftarrow \texttt{not}\ s$

$s \leftarrow \texttt{not}\ t$

$t \leftarrow \texttt{not}\ r$

$q \leftarrow \texttt{not}\ s \qquad p \leftarrow \texttt{not}\ r \qquad u \leftarrow \texttt{not}\ t$

$\neg q \leftarrow \texttt{not}\ s \qquad \neg p \leftarrow \texttt{not}\ r \qquad \neg u \leftarrow \texttt{not}\ t$

- culprit set: $\{r, s, t\}$

Scenario 3b – well-founded model & no answer set

Example (Scenario 3b)

$r \leftarrow \texttt{not}\ s$

$s \leftarrow \texttt{not}\ t$

$t \leftarrow \texttt{not}\ r$

$q \leftarrow \texttt{not}\ s$ $\quad$ $p \leftarrow \texttt{not}\ r$ $\quad$ $u \leftarrow \texttt{not}\ t$

$q' \leftarrow \texttt{not}\ s$ $\quad$ $p' \leftarrow \texttt{not}\ r$ $\quad$ $u' \leftarrow \texttt{not}\ t$

- culprit set: $\{r, s, t\}$

Scenario 3b – well-founded model & no answer set

Example (Scenario 3b)

$r \leftarrow \texttt{not}\ s$

$s \leftarrow \texttt{not}\ t$

$t \leftarrow \texttt{not}\ r$

$q \leftarrow \texttt{not}\ s$ $\quad$ $p \leftarrow \texttt{not}\ r$ $\quad$ $u \leftarrow \texttt{not}\ t$

$q' \leftarrow \texttt{not}\ s$ $\quad$ $p' \leftarrow \texttt{not}\ r$ $\quad$ $u' \leftarrow \texttt{not}\ t$

- culprit set: $\{r, s, t\}$
- $\mathcal{P}'$ no answer sets

Scenario 3b – well-founded model & no answer set

Example (Scenario 3b)

$r \leftarrow \texttt{not}\ s$

$s \leftarrow \texttt{not}\ t$

$t \leftarrow \texttt{not}\ r$

$q \leftarrow \texttt{not}\ s$ $\quad$ $p \leftarrow \texttt{not}\ r$ $\quad$ $u \leftarrow \texttt{not}\ t$

$q' \leftarrow \texttt{not}\ s$ $\quad$ $p' \leftarrow \texttt{not}\ r$ $\quad$ $u' \leftarrow \texttt{not}\ t$

- culprit set: $\{r, s, t\}$
- $\mathcal{P}'$ no answer sets
- 3-valued M-stable (regular) models: $\langle \emptyset, \emptyset \rangle$
  $\mathcal{U} = \{r, s, t, q, q', \ldots\}$

Inconsistency Scenarios

- $\mathcal{P}$ has no well-founded model
  1. the only answer set is $Lit_{\mathcal{P}}$
     $\Rightarrow$ **culprit set**: $\{a, \neg a\}$ both strictly derivable
     $\Rightarrow$ **reason**: explicit negation
  2. $\mathcal{P}$ has no answer sets
     $\Rightarrow$ **culprit set**: $\{a, \neg a\}$ one defeasibly derivable, other derivable
     $\Rightarrow$ **reason**: explicit negation & NAF
- $\mathcal{P}$ has a well-founded model, no answer sets
  - 3a $\mathcal{P}'$ has answer sets S_i
  - 3b $\mathcal{P}'$ has no answer sets

Inconsistency Scenarios

- $\mathcal{P}$ has no well-founded model
  1. the only answer set is $Lit_{\mathcal{P}}$
     $\Rightarrow$ **culprit set**: $\{a, \neg a\}$ both strictly derivable
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     $\Rightarrow$ **reason**: explicit negation & NAF
- $\mathcal{P}$ has a well-founded model, no answer sets
  - 3a $\mathcal{P}'$ has answer sets $S_i{}'$
  - 3b $\mathcal{P}'$ has no answer sets

Inconsistency Scenarios

- $\mathcal{P}$ has no well-founded model
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     $\Rightarrow$ **culprit set**: $\{a, \neg a\}$ one defeasibly derivable, other derivable
     $\Rightarrow$ **reason**: explicit negation & NAF
- $\mathcal{P}$ has a well-founded model, no answer sets
  - 3a $\mathcal{P}'$ has answer sets $S_i{}'$
    $\Rightarrow$ **culprit set**: $\{a_1, \neg a_1, \ldots, a_n, \neg a_n\}$ $a_i, a_i{}' \in S_i{}'$ one defeasibly derivable
  - 3b $\mathcal{P}'$ has no answer sets

Inconsistency Scenarios

- $\mathcal{P}$ has no well-founded model
  1. the only answer set is $Lit_{\mathcal{P}}$
     $\Rightarrow$ **culprit set**: $\{a, \neg a\}$ both strictly derivable
     $\Rightarrow$ **reason**: explicit negation
  2. $\mathcal{P}$ has no answer sets
     $\Rightarrow$ **culprit set**: $\{a, \neg a\}$ one defeasibly derivable, other derivable
     $\Rightarrow$ **reason**: explicit negation & NAF
- $\mathcal{P}$ has a well-founded model, no answer sets
  - 3a $\mathcal{P}'$ has answer sets ${S_i}'$
    $\Rightarrow$ **culprit set**: $\{a_1, \neg a_1, \ldots, a_n, \neg a_n\}$ $a_i, {a_i}' \in {S_i}'$ one defeasibly derivable
    $\Rightarrow$ **reason**: explicit negation & NAF
  - 3b $\mathcal{P}'$ has no answer sets

Inconsistency Scenarios

- $\mathcal{P}$ has no well-founded model
  1. the only answer set is $Lit_{\mathcal{P}}$
     $\Rightarrow$ **culprit set**: $\{a, \neg a\}$ both strictly derivable
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- $\mathcal{P}$ has a well-founded model, no answer sets
  - 3a $\mathcal{P}'$ has answer sets $S_i{}'$
    $\Rightarrow$ **culprit set**: $\{a_1, \neg a_1, \ldots, a_n, \neg a_n\}$ $a_i, a_i{}' \in S_i{}'$ one defeasibly derivable
    $\Rightarrow$ **reason**: explicit negation & NAF
  - 3b $\mathcal{P}'$ has no answer sets

Inconsistency Scenarios

- $\mathcal{P}$ has no well-founded model
  1. the only answer set is $Lit_{\mathcal{P}}$
     $\Rightarrow$ **culprit set**: $\{a, \neg a\}$ both strictly derivable
     $\Rightarrow$ **reason**: explicit negation
  2. $\mathcal{P}$ has no answer sets
     $\Rightarrow$ **culprit set**: $\{a, \neg a\}$ one defeasibly derivable, other derivable
     $\Rightarrow$ **reason**: explicit negation & NAF
- $\mathcal{P}$ has a well-founded model, no answer sets
  3a. $\mathcal{P}'$ has answer sets $S_i{}'$
     $\Rightarrow$ **culprit set**: $\{a_1, \neg a_1, \ldots, a_n, \neg a_n\}$ $a_i, a_i{}' \in S_i{}'$ one defeasibly derivable
     $\Rightarrow$ **reason**: explicit negation & NAF
  3b. $\mathcal{P}'$ has no answer sets
     $\Rightarrow$ **culprit set**: $\{b_1, \ldots, b_o\}$ odd negative dependency cycle

Inconsistency Scenarios

- $\mathcal{P}$ has no well-founded model
  1. the only answer set is $Lit_{\mathcal{P}}$
     $\Rightarrow$ **culprit set**: $\{a, \neg a\}$ both strictly derivable
     $\Rightarrow$ **reason**: explicit negation
  2. $\mathcal{P}$ has no answer sets
     $\Rightarrow$ **culprit set**: $\{a, \neg a\}$ one defeasibly derivable, other derivable
     $\Rightarrow$ **reason**: explicit negation & NAF
- $\mathcal{P}$ has a well-founded model, no answer sets
  3a. $\mathcal{P}'$ has answer sets $S_i{}'$
     $\Rightarrow$ **culprit set**: $\{a_1, \neg a_1, \ldots, a_n, \neg a_n\}$ $a_i, a_i{}' \in S_i{}'$ one defeasibly derivable
     $\Rightarrow$ **reason**: explicit negation & NAF
  3b. $\mathcal{P}'$ has no answer sets
     $\Rightarrow$ **culprit set**: $\{b_1, \ldots, b_o\}$ odd negative dependency cycle
     $\Rightarrow$ **reason**: NAF

Explaining Culprits

Inconsistency explanations

Example (Scenario 2)

..... some logic program .....

- culprit set: $\{q, \neg q\}$

Inconsistency explanations

Example (Scenario 2)

..... *some logic program* .....

- culprit set: $\{q, \neg q\}$

$$\mathcal{P}_2 \cup \{not\ r\} \vdash_{MP} q \qquad \mathcal{P}_2 \cup \{not\ p\} \vdash_{MP} \neg q$$

$$\uparrow$$

$$\mathcal{P}_2 \cup \{not\ \neg t\} \vdash_{MP} r$$

$$\uparrow$$

$$\mathcal{P}_2 \cup \emptyset \vdash_{MP} \neg t$$

Conclusion

Inconsistency – Contributions

Inconsistency scenarios:

1. ~~$S = Lit_{\mathcal{P}}$~~
2. ~~no answer set~~

$\Longrightarrow$ **4 scenarios** + **culprits**

Reasons for inconsistency:

1. explicit negation ($\neg$)
2. negation as failure (NAF) (`not`)

$+$ **explicit negation ($\neg$)** + **NAF (`not`)**

$+$ **explanations based on culprit sets**

Inconsistency – Contributions

Inconsistency scenarios:

1. ~~$S = Lit_P$~~
2. ~~no answer set~~

$\Longrightarrow$ **4 scenarios** + **culprits**

Reasons for inconsistency:

1. explicit negation ($\neg$)
2. negation as failure (NAF) (`not`)

$+$ **explicit negation ($\neg$)** + **NAF (`not`)**

$+$ **explanations based on culprit sets**

Appendix

Scenario 1 & 2 – no well-founded model

Scenario 1 – characterisation

If $\mathcal{P}$ has **no well-founded model** then:

the **only answer set is** $Lit_{\mathcal{P}}$
$\Leftrightarrow \mathcal{P} \vdash_{MP} a$ and $\mathcal{P} \vdash_{MP} \neg a$ (for some a)

$\{a, \neg a\}$ is a **culprit set** of $\mathcal{P}$
$\Leftrightarrow a, a' \in \mathcal{T}'_w$ and $\mathcal{P}' \vdash_{MP} a$ and $\mathcal{P}' \vdash_{MP} a'$

Scenario 2 – characterisation

If $\mathcal{P}$ has **no well-founded model** then:

$\mathcal{P}$ has **no answer set**
$\Leftrightarrow \nexists a$ with $\mathcal{P} \vdash_{MP} a$ and $\mathcal{P} \vdash_{MP} \neg a$

$\{a, \neg a\}$ is a **culprit set** of $\mathcal{P}$
$\Leftrightarrow a, a' \in \mathcal{T}'_w$ and a or a' is defeasibly derivable (in $\mathcal{P}'$)

Scenario 3 – well-founded model

Scenario 3a – characterisation

If $\mathcal{P}$ has a **well-founded model** then $\mathcal{P}$ has **no answer set** **and if** $\mathcal{P}'$ **has** n **answer sets** then

$\{a_1, \neg a_1, \ldots, a_n, \neg a_n\}$ is a **culprit set** of $\mathcal{P}$
$\Leftrightarrow$ $a_i, a_i' \in S_i'$ and a_i or a_i' is defeasibly derivable (in $\mathcal{P}'$)

Scenario 3b – characterisation

If $\mathcal{P}$ has a **well-founded model** then $\mathcal{P}$ has **no answer set** **and if** $\mathcal{P}'$ **has no answer set** then

$\{b_1, \ldots, b_o\}$ is a **culprit set** of $\mathcal{P}$
$\Leftrightarrow$ $b_1, \ldots, b_o, b_1$ is an odd negative dependency cycle in $\mathcal{P}'$ and $b_i \in \mathcal{U}_M'$

Scenario 2 – another example

Example (Scenario 2)

$q \leftarrow \mathit{not}\ r \qquad \neg q \leftarrow \neg s, \mathit{not}\ p \qquad r \leftarrow \mathit{not}\ \neg t \qquad \neg s \leftarrow \qquad \neg t \leftarrow$

Scenario 2 – another example

Example (Scenario 2)

$q \leftarrow not\ r \qquad \neg q \leftarrow \neg s, not\ p \qquad r \leftarrow not\ \neg t \qquad \neg s \leftarrow \qquad \neg t \leftarrow$

Scenario 2 – another example

Example (Scenario 2)

$q \leftarrow not\ r \qquad \neg q \leftarrow \neg s, not\ p \qquad r \leftarrow not\ \neg t \qquad \neg s \leftarrow \qquad \neg t \leftarrow$

Scenario 2 – another example

Example (Scenario 2)

$q \leftarrow not\ r \qquad \neg q \leftarrow \neg s, not\ p \qquad r \leftarrow not\ \neg t \qquad \neg s \leftarrow \qquad \neg t \leftarrow$

Scenario 2 – another example

Example (Scenario 2)

$q \leftarrow \mathit{not}\ r \qquad \neg q \leftarrow \neg s, \mathit{not}\ p \qquad r \leftarrow \mathit{not}\ \neg t \qquad \neg s \leftarrow \qquad \neg t \leftarrow$

Scenario 2 – another example

Example (Scenario 2)

$q \leftarrow \mathit{not}\ r \qquad \neg q \leftarrow \neg s, \mathit{not}\ p \qquad r \leftarrow \mathit{not}\ \neg t \qquad \neg s \leftarrow \qquad \neg t \leftarrow$

Scenario 2 – another example

Example (Scenario 2)

$q \leftarrow \mathit{not}\ r \qquad \neg q \leftarrow \neg s, \mathit{not}\ p \qquad r \leftarrow \mathit{not}\ \neg t \qquad \neg s \leftarrow \qquad \neg t \leftarrow$

Scenario 2 – another example

Example (Scenario 2)

$q \leftarrow \mathit{not}\ r \qquad \neg q \leftarrow \neg s, \mathit{not}\ p \qquad r \leftarrow \mathit{not}\ \neg t \qquad \neg s \leftarrow \qquad \neg t \leftarrow$

Scenario 2 – another example

Example (Scenario 2)

$q \leftarrow \mathit{not}\ r \quad \neg q \leftarrow \neg s, \mathit{not}\ p \quad r \leftarrow \mathit{not}\ \neg t \quad \neg s \leftarrow \quad \neg t \leftarrow$

- no well founded model, no answer set

Scenario 2 – another example

Example (Scenario 2)

$q \leftarrow \mathit{not}\ r \quad \neg q \leftarrow \neg s, \mathit{not}\ p \quad r \leftarrow \mathit{not}\ \neg t \quad \neg s \leftarrow \quad \neg t \leftarrow$

- no well founded model, no answer set
- well founded model of $\mathcal{P}'$: $\langle \{q, q', s', t'\}, \{p, r\} \rangle$

Scenario 2 – another example

Example (Scenario 2)

$q \leftarrow \mathit{not}\ r \quad \neg q \leftarrow \neg s, \mathit{not}\ p \quad r \leftarrow \mathit{not}\ \neg t \quad \neg s \leftarrow \quad \neg t \leftarrow$

- no well founded model, no answer set
- well founded model of $\mathcal{P}'$: $\langle \{q, q', s', t'\}, \{p, r\} \rangle$
- culprit sets: $\{q, \neg q\}$

Scenario 3b – another example

Example (Scenario 3b)

$$s \leftarrow w \qquad w \leftarrow \texttt{not } t \qquad t \leftarrow \neg x \qquad \neg x \leftarrow \texttt{not } \neg u$$

$$\neg u \leftarrow \texttt{not } v \qquad v \leftarrow \texttt{not } t, \texttt{not } x \qquad x \leftarrow \qquad y \leftarrow \texttt{not } x$$

- well-founded model: $\langle \{x\}, \{y\} \rangle$
- no answer set

Scenario 3b – another example

Example (Scenario 3b)

$s \leftarrow w$ $\quad w \leftarrow \texttt{not}\ t$ $\quad t \leftarrow x'$ $\quad x' \leftarrow \texttt{not}\ u'$

$u' \leftarrow \texttt{not}\ v$ $\quad v \leftarrow \texttt{not}\ t, \texttt{not}\ x$ $\quad x \leftarrow$ $\quad y \leftarrow \texttt{not}\ x$

Scenario 3b – another example

Example (Scenario 3b)

$s \leftarrow w$ $\quad w \leftarrow \texttt{not } t$ $\quad t \leftarrow x'$ $\quad x' \leftarrow \texttt{not } u'$

$u' \leftarrow \texttt{not } v$ $\quad v \leftarrow \texttt{not } t, \texttt{not } x$ $\quad x \leftarrow$ $\quad y \leftarrow \texttt{not } x$

- $\mathcal{P}'$ no answer sets

Scenario 3b – another example

Example (Scenario 3b)

$s \leftarrow w$ $\quad w \leftarrow \texttt{not}\ t$ $\quad t \leftarrow x'$ $\quad x' \leftarrow \texttt{not}\ u'$

$u' \leftarrow \texttt{not}\ v$ $\quad v \leftarrow \texttt{not}\ t, \texttt{not}\ x$ $\quad x \leftarrow$ $\quad y \leftarrow \texttt{not}\ x$

- $\mathcal{P}'$ no answer sets
- 3-valued M-stable models: $\langle \{x\}, \{y\} \rangle$,

Scenario 3b – another example

Example (Scenario 3b)

$s \leftarrow w$ $\quad w \leftarrow \texttt{not}\ t$ $\quad t \leftarrow x'$ $\quad x' \leftarrow \texttt{not}\ u'$

$u' \leftarrow \texttt{not}\ v$ $\quad v \leftarrow \texttt{not}\ t, \texttt{not}\ x$ $\quad x \leftarrow$ $\quad y \leftarrow \texttt{not}\ x$

- $\mathcal{P}'$ no answer sets
- 3-valued M-stable models: $\langle \{x\}, \{y\} \rangle$, $\mathcal{U}'_M = \{s, t, u', v, w, x'\}$

Scenario 3b – another example

Example (Scenario 3b)

$s \leftarrow w$ $\quad w \leftarrow \texttt{not } t$ $\quad t \leftarrow x'$ $\quad x' \leftarrow \texttt{not } u'$

$u' \leftarrow \texttt{not } v$ $\quad v \leftarrow \texttt{not } t, \texttt{not } x$ $\quad x \leftarrow$ $\quad y \leftarrow \texttt{not } x$

- 3-valued M-stable models: $\langle \{x\}, \{y\} \rangle$, $\mathcal{U}'_M = \{s, t, u', v, w, x'\}$
- negative dependency path: s

Scenario 3b – another example

Example (Scenario 3b)

$s \leftarrow w$ $\quad w \leftarrow \texttt{not } t$ $\quad t \leftarrow x'$ $\quad x' \leftarrow \texttt{not } u'$

$u' \leftarrow \texttt{not } v$ $\quad v \leftarrow \texttt{not } t, \texttt{not } x$ $\quad x \leftarrow$ $\quad y \leftarrow \texttt{not } x$

- 3-valued M-stable models: $\langle \{x\}, \{y\} \rangle$, $\mathcal{U}'_M = \{s, t, u', v, w, x'\}$
- negative dependency path: s, t

Scenario 3b – another example

Example (Scenario 3b)

$s \leftarrow w$ $\quad w \leftarrow \texttt{not}\ t$ $\quad t \leftarrow x'$ $\quad x' \leftarrow \texttt{not}\ u'$

$u' \leftarrow \texttt{not}\ v$ $\quad v \leftarrow \texttt{not}\ t, \texttt{not}\ x$ $\quad x \leftarrow$ $\quad y \leftarrow \texttt{not}\ x$

- 3-valued M-stable models: $\langle \{x\}, \{y\} \rangle$, $\mathcal{U}'_M = \{s, t, u', v, w, x'\}$
- negative dependency path: s, t

Scenario 3b – another example

Example (Scenario 3b)

$s \leftarrow w$ $\quad w \leftarrow \texttt{not } t$ $\quad t \leftarrow x'$ $\quad x' \leftarrow \texttt{not } u'$

$u' \leftarrow \texttt{not } v$ $\quad v \leftarrow \texttt{not } t, \texttt{not } x$ $\quad x \leftarrow$ $\quad y \leftarrow \texttt{not } x$

- 3-valued M-stable models: $\langle \{x\}, \{y\} \rangle$, $\mathcal{U}'_M = \{s, t, u', v, w, x'\}$
- negative dependency path: s, t, u'

Scenario 3b – another example

Example (Scenario 3b)

$s \leftarrow w$ $\quad w \leftarrow \texttt{not}\ t$ $\quad t \leftarrow x'$ $\quad x' \leftarrow \texttt{not}\ u'$

$u' \leftarrow \texttt{not}\ v$ $\quad v \leftarrow \texttt{not}\ t, \texttt{not}\ x$ $\quad x \leftarrow$ $\quad y \leftarrow \texttt{not}\ x$

- 3-valued M-stable models: $\langle \{x\}, \{y\} \rangle$, $\mathcal{U}'_M = \{s, t, u', v, w, x'\}$
- negative dependency path: s, t, u', v

Scenario 3b – another example

Example (Scenario 3b)

$s \leftarrow w$ $\quad w \leftarrow \texttt{not}\ t$ $\quad t \leftarrow x'$ $\quad x' \leftarrow \texttt{not}\ u'$

$u' \leftarrow \texttt{not}\ v$ $\quad v \leftarrow \texttt{not}\ t, \texttt{not}\ x$ $\quad x \leftarrow$ $\quad y \leftarrow \texttt{not}\ x$

- 3-valued M-stable models: $\langle \{x\}, \{y\} \rangle$, $\mathcal{U}'_M = \{s, t, u', v, w, x'\}$
- negative dependency path: s, t, u', v

Scenario 3b – another example

Example (Scenario 3b)

$s \leftarrow w$ $\quad w \leftarrow \texttt{not}\ t$ $\quad t \leftarrow x'$ $\quad x' \leftarrow \texttt{not}\ u'$

$u' \leftarrow \texttt{not}\ v$ $\quad v \leftarrow \texttt{not}\ t, \texttt{not}\ x$ $\quad x \leftarrow$ $\quad y \leftarrow \texttt{not}\ x$

- 3-valued M-stable models: $\langle \{x\}, \{y\} \rangle$, $\mathcal{U}'_M = \{s, t, u', v, w, x'\}$
- negative dependency path: s, t, u', v

Scenario 3b – another example

Example (Scenario 3b)

$s \leftarrow w$ $\quad w \leftarrow \texttt{not}\ t$ $\quad t \leftarrow x'$ $\quad x' \leftarrow \texttt{not}\ u'$

$u' \leftarrow \texttt{not}\ v$ $\quad v \leftarrow \texttt{not}\ t, \texttt{not}\ x$ $\quad x \leftarrow$ $\quad y \leftarrow \texttt{not}\ x$

- 3-valued M-stable models: $\langle \{x\}, \{y\} \rangle$, $\mathcal{U}'_M = \{s, t, u', v, w, x'\}$
- negative dependency path: s, t, u', v culprit set: $\{t, \neg u, v\}$

Inconsistency explanations

Example (Scenario 3a)

..... *some logic program* .....

- well-founded model, no answer sets, $\mathcal{P}'$ has answer sets

Inconsistency explanations

Example (Scenario 3a)

..... *some logic program* .....

- well-founded model, no answer sets, $\mathcal{P}'$ has answer sets
- check complementary literals in **answer sets** of $\mathcal{P}'$

Inconsistency explanations

Example (Scenario 3a)

..... *some logic program* .....

- well-founded model, no answer sets, $\mathcal{P}'$ has answer sets
- check complementary literals in **answer sets** of $\mathcal{P}'$
- culprit sets: $\{p, \neg p, q, \neg q\}$

Inconsistency explanations

Example (Scenario 3a)

..... some logic program .....

- well-founded model, no answer sets, $\mathcal{P}'$ has answer sets
- check complementary literals in **answer sets** of $\mathcal{P}'$
- culprit sets: $\{p, \neg p, q, \neg q\}$

$$\mathcal{P}_3 \cup \{not\ s\} \vdash_{MP} q$$
$$\uparrow$$
$$\mathcal{P}_3 \cup \{not\ r\} \vdash_{MP} s$$
$$\uparrow$$
$$\mathcal{P}_3 \cup \{not\ s\} \vdash_{MP} r$$
$$\uparrow$$
$$\mathcal{P}_3 \cup \{not\ r\} \vdash_{MP} s$$
$$\vdots$$

$$\mathcal{P}_3 \cup \{not\ s\} \vdash_{MP} \neg q$$
$$\uparrow$$
$$\mathcal{P}_3 \cup \{not\ r\} \vdash_{MP} s$$
$$\uparrow$$
$$\mathcal{P}_3 \cup \{not\ s\} \vdash_{MP} r$$
$$\uparrow$$
$$\mathcal{P}_3 \cup \{not\ r\} \vdash_{MP} s$$
$$\vdots$$

Inconsistency Explanations

Example (Scenario 3b)

..... *some logic program* .....

- well-founded model, no answer sets, $\mathcal{P}'$ has no answer sets

Inconsistency Explanations

Example (Scenario 3b)

..... *some logic program* .....

- well-founded model, no answer sets, $\mathcal{P}'$ has no answer sets
- check **negative dependency cycle** in $\mathcal{P}'$

Inconsistency Explanations

Example (Scenario 3b)

..... *some logic program* .....

- well-founded model, no answer sets, $\mathcal{P}'$ has no answer sets
- check **negative dependency cycle** in $\mathcal{P}'$
- culprit sets: $\{\neg u, v, t\}$

Inconsistency Explanations

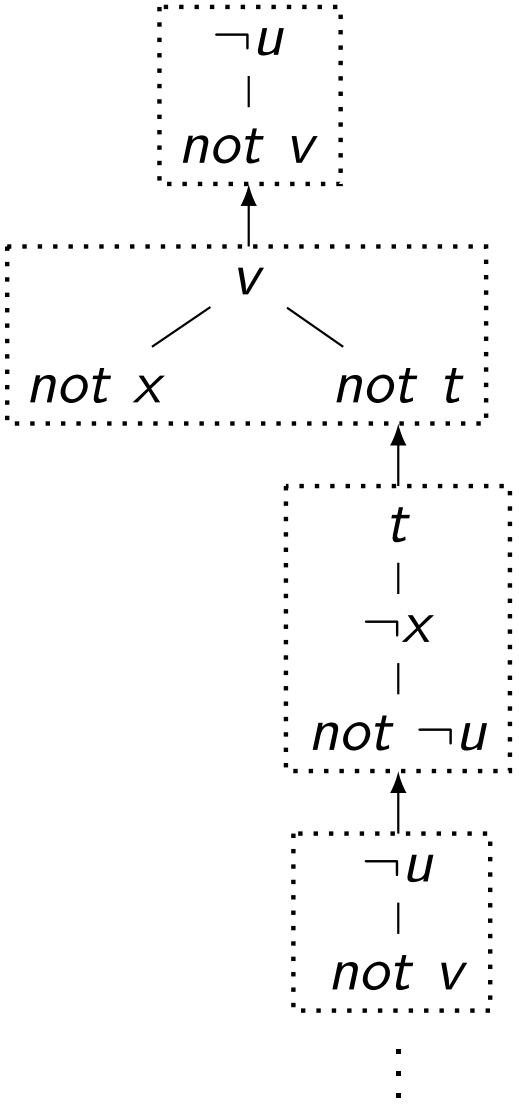
Example (Scenario 3b)

..... *some logic program* .....

- well-founded model, no answer sets, $\mathcal{P}'$ has no answer sets
- check **negative dependency cycle** in $\mathcal{P}'$
- culprit sets: $\{\neg u, v, t\}$

$$\mathcal{P}_4 \cup \{not\ v\} \vdash_{MP} \neg u$$
$$\uparrow$$
$$\mathcal{P}_4 \cup \{not\ t, not\ x\} \vdash_{MP} v$$
$$\uparrow$$
$$\mathcal{P}_4 \cup \{not\ \neg u\} \vdash_{MP} t$$
$$\uparrow$$
$$\mathcal{P}_4 \cup \{not\ v\} \vdash_{MP} \neg u$$
$$\vdots$$

Inconsistency Explanations



Inconsistency Explanations