

Pocket calculators for hard combinatorial search and optimization problems

Torsten Schaub

University of Potsdam



Outline

1 Introduction

2 Modeling

3 Solving

4 Summary

Outline

1 Introduction

2 Modeling

3 Solving

4 Summary

Characteristics of good pocket calculators

- 0 handy
- 1 easy to use
- 2 lots of operations
- 3 computes effectively

Claim

Answer Set Programming (ASP) offers good pocket calculators for hard combinatorial search and optimization problems

Characteristics of good pocket calculators

- 0 handy
- 1 easy to use
- 2 lots of operations
- 3 computes effectively

Claim

Answer Set Programming (ASP) offers good pocket calculators for hard combinatorial search and optimization problems

Characteristics of good pocket calculators

- 0 handy
- 1 easy to use
- 2 lots of operations
- 3 computes effectively

ASP may run on your cell phone

Claim

Answer Set Programming (ASP) offers good pocket calculators for hard combinatorial search and optimization problems

Characteristics of good pocket calculators

- 0 handy ✓
- 1 easy to use
- 2 lots of operations
- 3 computes effectively

ASP may run on your cell phone

Claim

Answer Set Programming (ASP) offers good pocket calculators for hard combinatorial search and optimization problems

Characteristics of good pocket calculators

0 handy ✓

ASP may run on your cell phone

1 easy to use

ASP has a high-level modeling language

2 lots of operations

3 computes effectively

Claim

Answer Set Programming (ASP) offers good pocket calculators for hard combinatorial search and optimization problems

Characteristics of good pocket calculators

- 0 handy ✓
- 1 easy to use ✓
- 2 lots of operations
- 3 computes effectively

ASP may run on your cell phone

ASP has a high-level modeling language

Claim

Answer Set Programming (ASP) offers good pocket calculators for hard combinatorial search and optimization problems

Characteristics of good pocket calculators

0 handy ✓

ASP may run on your cell phone

1 easy to use ✓

ASP has a high-level modeling language

2 lots of operations

ASP offers various reasoning modes

3 computes effectively

Claim

Answer Set Programming (ASP) offers good pocket calculators for hard combinatorial search and optimization problems

Characteristics of good pocket calculators

- 0 handy ✓
- 1 easy to use ✓
- 2 lots of operations ✓
- 3 computes effectively

ASP may run on your cell phone

ASP has a high-level modeling language

ASP offers various reasoning modes

Claim

Answer Set Programming (ASP) offers good pocket calculators for hard combinatorial search and optimization problems

Characteristics of good pocket calculators

- 0 handy ✓
- 1 easy to use ✓
- 2 lots of operations ✓
- 3 computes effectively

ASP may run on your cell phone

ASP has a high-level modeling language

ASP offers various reasoning modes

ASP solvers are highly effective

Claim

Answer Set Programming (ASP) offers good pocket calculators for hard combinatorial search and optimization problems

Characteristics of good pocket calculators

- 0 handy ✓ *ASP may run on your cell phone*
- 1 easy to use ✓ *ASP has a high-level modeling language*
- 2 lots of operations ✓ *ASP offers various reasoning modes*
- 3 computes effectively ✓ *ASP solvers are highly effective*

Claim

Answer Set Programming (ASP) offers good pocket calculators for hard combinatorial search and optimization problems

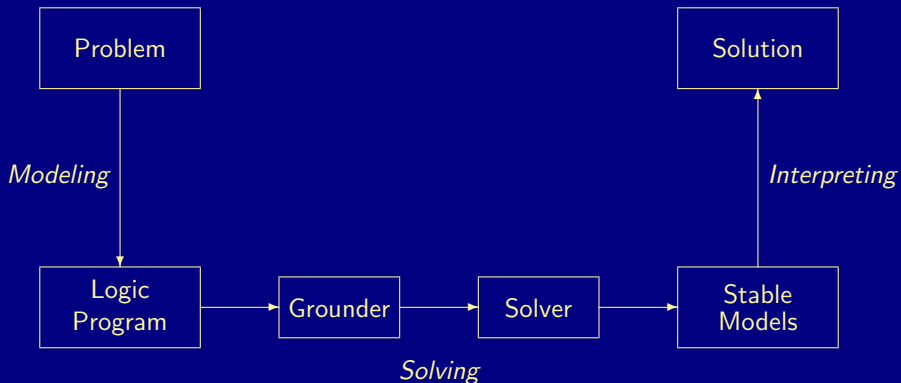
Characteristics of good pocket calculators

- 0 handy ✓ *ASP may run on your cell phone*
- 1 easy to use ✓ *ASP has a high-level modeling language*
- 2 lots of operations ✓ *ASP offers various reasoning modes*
- 3 computes effectively ✓ *ASP solvers are highly effective*

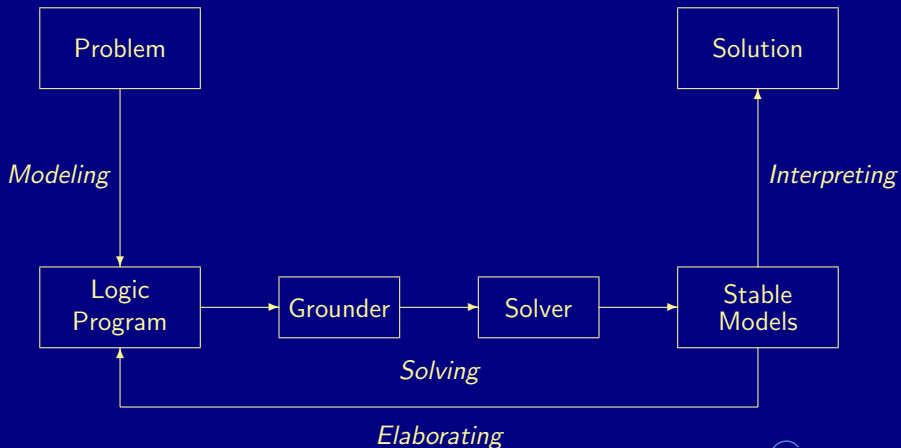
Claim

Answer Set Programming (ASP) offers good pocket calculators for hard combinatorial search and optimization problems ✓

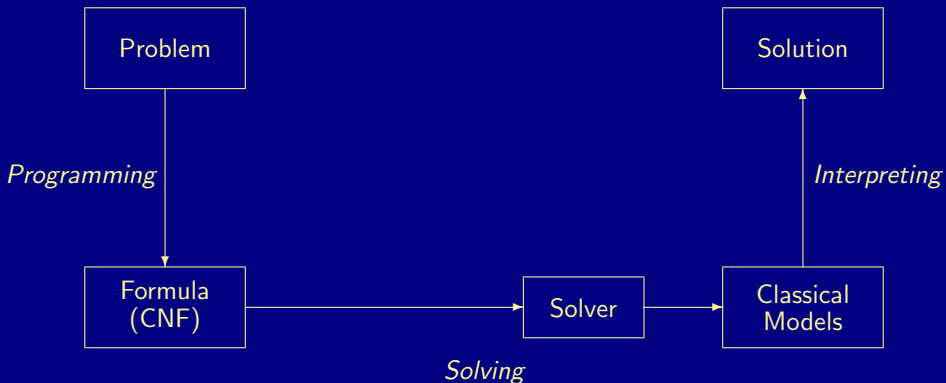
ASP solving



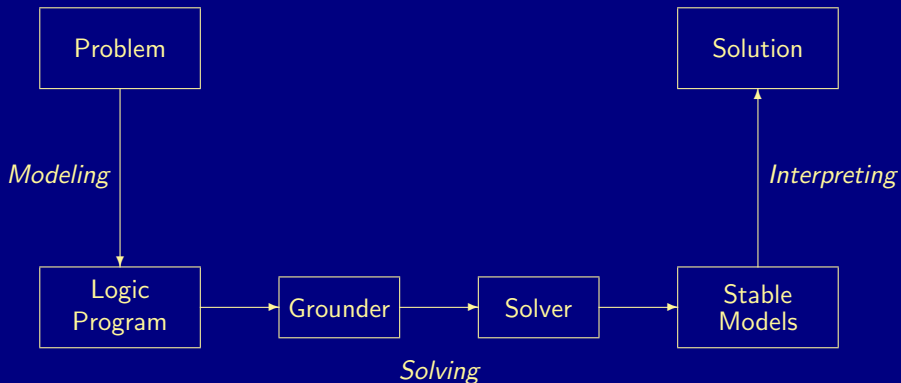
ASP solving



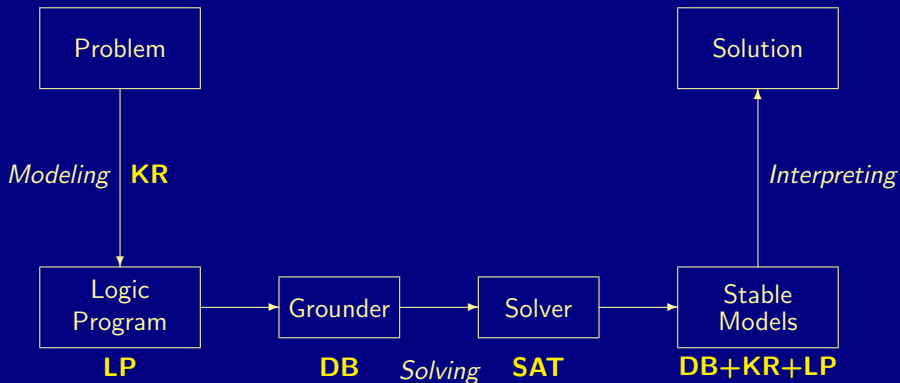
SAT solving



Rooting ASP solving



Rooting ASP solving



Outline

1 Introduction

2 Modeling

3 Solving

4 Summary

Language constructs

- Variables

$$p(X) \text{ :- } q(X)$$

- Conditional literals

$$p \text{ :- } q(X) \text{ : } r(X)$$

- Disjunction

$$p(X) \text{ ; } q(X) \text{ :- } r(X)$$

- Integrity constraints

$$\text{:- } q(X), p(X)$$

- Choice

$$2 \{ p(X,Y) \text{ : } q(X) \} 7 \text{ :- } r(Y)$$

- Aggregates

$$s(Y) \text{ :- } r(Y), 2 \text{ \#sum} \{ X \text{ : } p(X,Y), q(X) \} 7$$

- Optimization

$$\text{:- } \sim q(X), p(X,C) [C]$$

$$\text{\#minimize } \{ C \text{ : } q(X), p(X,C) \}$$

Language constructs

- Variables

$$p(X) \text{ :- } q(X)$$

- Conditional literals

$$p \text{ :- } q(X) \text{ : } r(X)$$

- Disjunction

$$p(X) \text{ ; } q(X) \text{ :- } r(X)$$

- Integrity constraints

$$\text{:- } q(X), p(X)$$

- Choice

$$2 \{ p(X,Y) \text{ : } q(X) \} 7 \text{ :- } r(Y)$$

- Aggregates

$$s(Y) \text{ :- } r(Y), 2 \#sum\{ X \text{ : } p(X,Y), q(X) \} 7$$

- Optimization

$$\text{:- } \sim q(X), p(X,C) [C]$$

$$\#minimize \{ C \text{ : } q(X), p(X,C) \}$$

Language constructs

- Variables

$$p(X) \text{ :- } q(X)$$

- Conditional literals

$$p \text{ :- } q(X) \text{ : } r(X)$$

- Disjunction

$$p(X) \text{ ; } q(X) \text{ :- } r(X)$$

- Integrity constraints

$$\text{:- } q(X), p(X)$$

- Choice

$$2 \{ p(X,Y) \text{ : } q(X) \} 7 \text{ :- } r(Y)$$

- Aggregates

$$s(Y) \text{ :- } r(Y), 2 \text{ \#sum} \{ X \text{ : } p(X,Y), q(X) \} 7$$

- Optimization

$$\text{:- } \sim q(X), p(X,C) [C]$$

$$\text{\#minimize } \{ C \text{ : } q(X), p(X,C) \}$$

Language constructs

- Variables

$$p(X) \text{ :- } q(X)$$

- Conditional literals

$$p \text{ :- } q(X) \text{ : } r(X)$$

- Disjunction

$$p(X) \text{ ; } q(X) \text{ :- } r(X)$$

- Integrity constraints

$$\text{:- } q(X), p(X)$$

- Choice

$$2 \{ p(X,Y) \text{ : } q(X) \} 7 \text{ :- } r(Y)$$

- Aggregates

$$s(Y) \text{ :- } r(Y), 2 \#sum\{ X \text{ : } p(X,Y), q(X) \} 7$$

- Optimization

$$\text{:- } \sim q(X), p(X,C) [C]$$

$$\#minimize \{ C \text{ : } q(X), p(X,C) \}$$

Language constructs

- Variables

$$p(X) \text{ :- } q(X)$$

- Conditional literals

$$p \text{ :- } q(X) \text{ : } r(X)$$

- Disjunction

$$p(X) \text{ ; } q(X) \text{ :- } r(X)$$

- Integrity constraints

$$\text{:- } q(X), p(X)$$

- Choice

$$2 \{ p(X,Y) \text{ : } q(X) \} 7 \text{ :- } r(Y)$$

- Aggregates

$$s(Y) \text{ :- } r(Y), 2 \text{ \#sum} \{ X \text{ : } p(X,Y), q(X) \} 7$$

- Optimization

$$\text{:- } \sim q(X), p(X,C) [C]$$

$$\text{\#minimize } \{ C \text{ : } q(X), p(X,C) \}$$

Language constructs

- Variables $p(X) \text{ :- } q(X)$
- Conditional literals $p \text{ :- } q(X) \text{ : } r(X)$
- Disjunction $p(X) \text{ ; } q(X) \text{ :- } r(X)$
- Integrity constraints $\text{:- } q(X), p(X)$
- Choice $2 \{ p(X,Y) \text{ : } q(X) \} 7 \text{ :- } r(Y)$
- Aggregates $s(Y) \text{ :- } r(Y), 2 \#sum\{ X \text{ : } p(X,Y), q(X) \} 7$
- Optimization $\text{:- } \sim q(X), p(X,C) [C]$
 $\#minimize \{ C \text{ : } q(X), p(X,C) \}$

Language constructs

- Variables

$$p(X) \text{ :- } q(X)$$

- Conditional literals

$$p \text{ :- } q(X) \text{ : } r(X)$$

- Disjunction

$$p(X) \text{ ; } q(X) \text{ :- } r(X)$$

- Integrity constraints

$$\text{:- } q(X), p(X)$$

- Choice

$$2 \{ p(X,Y) \text{ : } q(X) \} 7 \text{ :- } r(Y)$$

- Aggregates

$$s(Y) \text{ :- } r(Y), 2 \text{ \#sum} \{ X \text{ : } p(X,Y), q(X) \} 7$$

- Optimization

$$\text{:- } \sim q(X), p(X,C) [C]$$

$$\text{\#minimize } \{ C \text{ : } q(X), p(X,C) \}$$

Language constructs

- Variables $p(X) :- q(X)$
- Conditional literals $p :- q(X) : r(X)$
- Disjunction $p(X) ; q(X) :- r(X)$
- Integrity constraints $:- q(X), p(X)$
- Choice $2 \{ p(X,Y) : q(X) \} 7 :- r(Y)$
- Aggregates $s(Y) :- r(Y), 2 \#sum\{ X : p(X,Y), q(X) \} 7$
- Optimization
 - Weak constraints $:\sim q(X), p(X,C) [C]$
 - Statements $\#minimize \{ C : q(X), p(X,C) \}$

Language constructs

- Variables $p(X) :- q(X)$
- Conditional literals $p :- q(X) : r(X)$
- Disjunction $p(X) ; q(X) :- r(X)$
- Integrity constraints $:- q(X), p(X)$
- Choice $2 \{ p(X,Y) : q(X) \} 7 :- r(Y)$
- Aggregates $s(Y) :- r(Y), 2 \#sum\{ X : p(X,Y), q(X) \} 7$
- Optimization
 - Weak constraints $:\sim q(X), p(X,C) [C]$
 - Statements $\#minimize \{ C : q(X), p(X,C) \}$

Language constructs

- Variables $p(X) \text{ :- } q(X)$
- Conditional literals $p \text{ :- } q(X) \text{ : } r(X)$
- Disjunction $p(X) \text{ ; } q(X) \text{ :- } r(X)$
- Integrity constraints $\text{ :- } q(X), p(X)$
- Choice $2 \{ p(X,Y) \text{ : } q(X) \} 7 \text{ :- } r(Y)$
- Aggregates $s(Y) \text{ :- } r(Y), 2 \text{ \#sum}\{ X \text{ : } p(X,Y), q(X) \} 7$
- Optimization
 - Weak constraints $\text{ :} \sim q(X), p(X,C) [C]$
 - Statements $\text{ \#minimize } \{ C \text{ : } q(X), p(X,C) \}$

Language constructs

- Variables $p(X) :- q(X)$
- Conditional literals $p :- q(X) : r(X)$
- Disjunction $p(X) ; q(X) :- r(X)$
- Integrity constraints $:- q(X), p(X)$
- Choice $2 \{ p(X,Y) : q(X) \} 7 :- r(Y)$
- Aggregates $s(Y) :- r(Y), 2 \#sum\{ X : p(X,Y), q(X) \} 7$
- Multi-objective optimization
 - Weak constraints $:\sim q(X), p(X,C) [C@42]$
 - Statements $\#minimize \{ C@42 : q(X), p(X,C) \}$

Basic methodology

Methodology

Generate and Test (or: Guess and Check)

Generator Generate potential stable model candidates
(typically through non-deterministic constructs)

Tester Eliminate invalid candidates
(typically through integrity constraints)

Peanutshell

Logic program = Data + Generator + Tester (+ Optimizer)

Basic methodology

Methodology

Generate and Test (or: Guess and Check)

Generator Generate potential stable model candidates
(typically through non-deterministic constructs)

Tester Eliminate invalid candidates
(typically through integrity constraints)

Peanutshell

Logic program = Data + Generator + Tester (+ Optimizer)

Satisfiability testing

$$(a \leftrightarrow b) \wedge c$$

Satisfiability testing

$$(a \leftrightarrow b) \wedge c$$

```
{ a ; b ; c }.
```

```
:- not a, b.
```

```
:- a, not b.
```

```
:- not c.
```

Maximum satisfiability testing

“($a \leftrightarrow b$) \wedge c ”

```
{ a ; b ; c }.
```

```
:- not a, b.
```

```
:- a, not b.
```

```
:- not c.
```

```
{ a ; b ; c }.
```

```
:- not a, b.
```

```
:~ a, not b. [10@2]
```

```
:~ not c. [100@1]
```

n-Queens

Basic encoding

```
{ queen(1..n,1..n) }.
```

```
:- { queen(I,J) } != n.
```

```
:- queen(I,J), queen(I,JJ), J != JJ.
```

```
:- queen(I,J), queen(II,J), I != II.
```

```
:- queen(I,J), queen(II,JJ), (I,J) != (II,JJ), I-J = II-JJ.
```

```
:- queen(I,J), queen(II,JJ), (I,J) != (II,JJ), I+J = II+JJ.
```

n-Queens

Advanced encoding

```
{ queen(I,1..n) } = 1 :- I = 1..n.
```

```
{ queen(1..n,J) } = 1 :- J = 1..n.
```

```
:- { queen(D-J,J) } > 1, D = 2..2*n.
```

```
:- { queen(D+J,J) } > 1, D = 1-n..n-1.
```

Traveling salesperson

Basic encoding

```

1 { cycle(X,Y) : edge(X,Y) } 1 :- node(X).
1 { cycle(X,Y) : edge(X,Y) } 1 :- node(Y).

reached(X) :- X = #min { Y : node(Y) }.
reached(Y) :- cycle(X,Y), reached(X).

:- node(Y), not reached(Y).

#minimize { C,X,Y : cycle(X,Y), cost(X,Y,C) }.

node(X) :- edge(X,_).
node(X) :- edge(_,X).

edge(1,2).   edge(1,3).   edge(1,4).
edge(2,4).   edge(2,5).   edge(2,6).   [...]

```

Traveling salesperson

Basic encoding

```

1 { cycle(X,Y) : edge(X,Y) } 1 :- node(X).
1 { cycle(X,Y) : edge(X,Y) } 1 :- node(Y).

reached(X) :- X = #min { Y : node(Y) }.
reached(Y) :- cycle(X,Y), reached(X).

:- node(Y), not reached(Y).

#minimize { C,X,Y : cycle(X,Y), cost(X,Y,C) }.

node(X) :- edge(X,_).
node(X) :- edge(_,X).

edge(1,2).   edge(1,3).   edge(1,4).
edge(2,4).   edge(2,5).   edge(2,6).   [...]

```


Traveling salesperson

Basic encoding

```

1 { cycle(X,Y) : edge(X,Y) } 1 :- node(X).
1 { cycle(X,Y) : edge(X,Y) } 1 :- node(Y).

reached(X) :- X = #min { Y : node(Y) }.
reached(Y) :- cycle(X,Y), reached(X).

:- node(Y), not reached(Y).

#minimize { C,X,Y : cycle(X,Y), cost(X,Y,C) }.

node(X) :- edge(X,_).
node(X) :- edge(_,X).

edge(1,2).   edge(1,3).   edge(1,4).
edge(2,4).   edge(2,5).   edge(2,6).   [...]

```

Company Controls

```
controls(X,Y) :-
    #sum+ { S: owns(X,Y,S);
           S,Z: controls(X,Z), owns(Z,Y,S) } > 50,
    company(X), company(Y), X != Y.
```

```
company(c_1).    owns(c_1,c_2,60).
                 owns(c_1,c_3,20).
company(c_2).    owns(c_2,c_3,35).
company(c_3).    owns(c_3,c_4,51).
company(c_4).
```

Outline

1 Introduction

2 Modeling

3 Solving

4 Summary

Reasoning modes

- ASP solvers offer
 - Satisfiability testing
 - Enumeration
 - Projection
 - Intersection
 - Union
 - Optimization
 - and combinations of them

For instance, *clasp* allows for

ASP solving (*smodels* format)

MaxSAT and SAT solving (extended *dimacs* format)

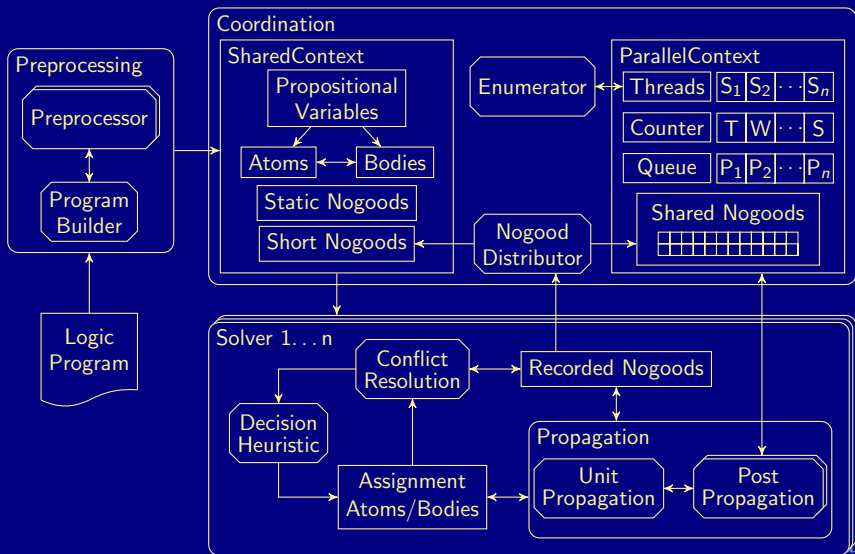
PB solving (*opb* and *wbo* format)

Reasoning modes

- ASP solvers offer
 - Satisfiability testing
 - Enumeration
 - Projection
 - Intersection
 - Union
 - Optimization
 - and combinations of them
- For instance, *clasp* allows for
 - ASP solving (*smodels* format)
 - MaxSAT and SAT solving (extended *dimacs* format)
 - PB solving (*opb* and *wbo* format)

Reasoning modes

- ASP solvers offer
 - Satisfiability testing
 - Enumeration
 - Projection
 - Intersection
 - Union
 - Optimization
 - and combinations of them
- For instance, *clasp* allows for
 - ASP solving (*smodels* format)
 - MaxSAT and SAT solving (extended *dimacs* format)
 - PB solving (*opb* and *wbo* format)

Multi-threaded architecture of *clasp*

Outline

1 Introduction

2 Modeling

3 Solving

4 Summary

Well then ...?

Claim

ASP offers good pocket calculators for hard combinatorial search and optimization problems

- | | | | |
|---|----------------------|---|---|
| 0 | handy | <i>ASP may run on your cell phone</i> | ✗ |
| 1 | easy to use | <i>ASP has a high-level modeling language</i> | ✓ |
| 2 | lots of operations | <i>ASP offers various reasoning modes</i> | ✓ |
| 3 | computes effectively | <i>ASP solvers are highly effective</i> | ✓ |

Well then ...?

Claim

ASP offers good pocket calculators for hard combinatorial search and optimization problems

- | | | | |
|---|----------------------|---|---|
| 0 | handy | <i>ASP may run on your cell phone</i> | ✗ |
| 1 | easy to use | <i>ASP has a high-level modeling language</i> | ✓ |
| 2 | lots of operations | <i>ASP offers various reasoning modes</i> | ✓ |
| 3 | computes effectively | <i>ASP solvers are highly effective</i> | ✓ |

Well then ...?

Claim

ASP offers good pocket calculators for hard combinatorial search and optimization problems ✓

- | | | | |
|---|----------------------|---|---|
| 0 | handy | <i>ASP may run on your cell phone</i> | ✗ |
| 1 | easy to use | <i>ASP has a high-level modeling language</i> | ✓ |
| 2 | lots of operations | <i>ASP offers various reasoning modes</i> | ✓ |
| 3 | computes effectively | <i>ASP solvers are highly effective</i> | ✓ |

ASPyyclopedia

■ Systems

- asperix <http://www.info.univ-angers.fr/pub/claire/asperix>
- assat <http://assat.cs.ust.hk>
- clasp, gringo, clingo, etc. <http://potassco.sourceforge.net>
- cmodels <http://www.cs.utexas.edu/users/tag/cmodels>
- dlv <http://www.dlvsystem.com>
- lp2* <http://research.ics.aalto.fi/software/asp>
- smodels, lparse, gnt <http://www.tcs.hut.fi/Software>
- wasp <https://www.mat.unical.it/ricca/wasp>
- sup <http://www.cs.utexas.edu/users/tag/sup>

■ User's guides

- DLV Systems
http://www.dlvsystem.com/html/DLV_User_Manual.html
- Potassco
<http://sourceforge.net/projects/potassco/files/guide>

■ Literature [1, 6, 8, 14], [13, 7, 4, 3], [10, 11, 2, 16, 15, 12, 9, 5], etc.



ASPyyclopedia

■ Systems — *best suited for beginners*

■ clingo <http://potassco.sourceforge.net>

■ dlvs <http://www.dlvsystem.com>

■ User's guides

■ DLV Systems

http://www.dlvsystem.com/html/DLV_User_Manual.html

■ Potassco

<http://sourceforge.net/projects/potassco/files/guide>

■ Literature [1, 6, 8, 14], [13, 7, 4, 3], [10, 11, 2, 16, 15, 12, 9, 5], etc.



ASPyyclopedia

■ Systems

- asperix <http://www.info.univ-angers.fr/pub/claire/asperix>
- assat <http://assat.cs.ust.hk>
- clasp, gringo, clingo, etc. <http://potassco.sourceforge.net>
- cmodels <http://www.cs.utexas.edu/users/tag/cmodels>
- dlv <http://www.dlvsystem.com>
- lp2* <http://research.ics.aalto.fi/software/asp>
- smodels, lparse, gnt <http://www.tcs.hut.fi/Software>
- wasp <https://www.mat.unical.it/ricca/wasp>
- sup <http://www.cs.utexas.edu/users/tag/sup>

■ User's guides

- DLV Systems
http://www.dlvsystem.com/html/DLV_User_Manual.html
- Potassco
<http://sourceforge.net/projects/potassco/files/guide>

■ Literature [1, 6, 8, 14], [13, 7, 4, 3], [10, 11, 2, 16, 15, 12, 9, 5], etc.



ASPyyclopedia

■ Systems

- asperix <http://www.info.univ-angers.fr/pub/claire/asperix>
- assat <http://assat.cs.ust.hk>
- clasp, gringo, clingo, etc. <http://potassco.sourceforge.net>
- cmodels <http://www.cs.utexas.edu/users/tag/cmodels>
- dlv <http://www.dlvsystem.com>
- lp2* <http://research.ics.aalto.fi/software/asp>
- smodels, lparse, gnt <http://www.tcs.hut.fi/Software>
- wasp <https://www.mat.unical.it/ricca/wasp>
- sup <http://www.cs.utexas.edu/users/tag/sup>

■ User's guides

- DLV Systems
http://www.dlvsystem.com/html/DLV_User_Manual.html
- Potassco
<http://sourceforge.net/projects/potassco/files/guide>

- Literature [1, 6, 8, 14], [13, 7, 4, 3], [10, 11, 2, 16, 15, 12, 9, 5], etc.



ASPylopedia

■ Systems

- asperix <http://www.info.univ-angers.fr/pub/claire/asperix>
- assat <http://assat.cs.ust.hk>
- clasp, gringo, clingo, etc. <http://potassco.sourceforge.net>
- cmodels <http://www.cs.utexas.edu/users/tag/cmodels>
- dlv <http://www.dlvsystem.com>
- lp2* <http://research.ics.aalto.fi/software/asp>
- smodels, lparse, gnt <http://www.tcs.hut.fi/Software>
- wasp <https://www.mat.unical.it/ricca/wasp>
- sup <http://www.cs.utexas.edu/users/tag/sup>

■ User's guides

- DLV Systems
http://www.dlvsystem.com/html/DLV_User_Manual.html
- Potassco
<http://sourceforge.net/projects/potassco/files/guide>

■ Literature [1, 6, 8, 14], [13, 7, 4, 3], [10, 11, 2, 16, 15, 12, 9, 5], etc.



- [1] C. Baral.
Knowledge Representation, Reasoning and Declarative Problem Solving.
Cambridge University Press, 2003.
- [2] C. Baral and M. Gelfond.
Logic programming and knowledge representation.
Journal of Logic Programming, 12:1–80, 1994.
- [3] G. Brewka, T. Eiter, and M. Truszczyński.
Answer set programming at a glance.
Communications of the ACM, 54(12):92–103, 2011.
- [4] T. Eiter, G. Ianni, and T. Krennwallner.
Answer Set Programming: A Primer.
In S. Tessaris, E. Franconi, T. Eiter, C. Gutierrez, S. Handschuh, M. Rousset, and R. Schmidt, editors, *Fifth International Reasoning Web Summer School (RW'09)*, volume 5689 of *Lecture Notes in Computer Science*, pages 40–110. Springer-Verlag, 2009.

- [5] M. Gebser, A. Harrison, R. Kaminski, V. Lifschitz, and T. Schaub.
Abstract Gringo.
Theory and Practice of Logic Programming, 15(4-5):449–463, 2015.
Available at <http://arxiv.org/abs/1507.06576>.
- [6] M. Gebser, R. Kaminski, B. Kaufmann, and T. Schaub.
Answer Set Solving in Practice.
Synthesis Lectures on Artificial Intelligence and Machine Learning.
Morgan and Claypool Publishers, 2012.
- [7] M. Gelfond.
Answer sets.
In V. Lifschitz, F. van Harmelen, and B. Porter, editors, *Handbook of Knowledge Representation*, chapter 7, pages 285–316. Elsevier Science, 2008.
- [8] M. Gelfond and Y. Kahl.
Knowledge Representation, Reasoning, and the Design of Intelligent Agents: The Answer-Set Programming Approach.

Cambridge University Press, 2014.

- [9] M. Gelfond and N. Leone.
Logic programming and knowledge representation — the A-Prolog perspective.
Artificial Intelligence, 138(1-2):3–38, 2002.
- [10] M. Gelfond and V. Lifschitz.
The stable model semantics for logic programming.
In R. Kowalski and K. Bowen, editors, *Proceedings of the Fifth International Conference and Symposium of Logic Programming (ICLP'88)*, pages 1070–1080. MIT Press, 1988.
- [11] M. Gelfond and V. Lifschitz.
Logic programs with classical negation.
In D. Warren and P. Szeredi, editors, *Proceedings of the Seventh International Conference on Logic Programming (ICLP'90)*, pages 579–597. MIT Press, 1990.
- [12] V. Lifschitz.

Answer set programming and plan generation.

Artificial Intelligence, 138(1-2):39–54, 2002.

[13] V. Lifschitz.

Introduction to answer set programming.

Unpublished draft, 2004.

[14] V. Marek and M. Truszczyński.

Nonmonotonic logic: context-dependent reasoning.

Artificial Intelligence. Springer-Verlag, 1993.

[15] V. Marek and M. Truszczyński.

Stable models and an alternative logic programming paradigm.

In K. Apt, V. Marek, M. Truszczyński, and D. Warren, editors, *The Logic Programming Paradigm: a 25-Year Perspective*, pages 375–398.

Springer-Verlag, 1999.

[16] I. Niemelä.

Logic programs with stable model semantics as a constraint programming paradigm.

Annals of Mathematics and Artificial Intelligence, 25(3-4):241–273,
1999.