Algorithmic Decision Theory meets Logic

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Thanks to the ADT-15 and LPNMR-15 organizers and chairs.

One talk, two plans

- Common appetizer

ADT plan

- Fair division
- Coalition structure formation
- Combinatorial auctions
- Multiple referenda
- Committee elections
- Multiattribute decision making
- Voting under uncertainty

LPNMR plan

- Propositional Logic
- MAXSAT
- Default Logic
- Weighted Goals
- Prioritized Goals
- Preference Logics
- Nonmonotonic Preferences

- Common dessert: ASP and ADT, a love match

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Algorithmic Decision Theory

for logic programmers and nonmonotonic logicians

Design and study of languages and computational methods for expressing and solving decision making problems, such as

sequential decision making	resource allocation
multiattribute decision making	strategic games
coalition structure formation	decision under uncertainty
committee elections	multiple referenda
recommender systems	(and more)

- Domains of solutions in algorithmic decision theory often have a *combinatorial* structure
$$A = D_1 \times \ldots \times D_p$$
where D_i = finite set of values associated with a variable X_i.
- Algorithmic decision theory is computationally hard.

Logic in Artificial Intelligence
for algorithmic decision theorists

Two distinct roles:

- a *declarative representation language*
  - rich expressivity of logics → representing complex problems
- a *generic problem solving tool*
  - SAT (satisfiability) solvers
  - QBF (quantified Boolean formulas)
  - the early stage: Prolog
  - the modern stage: ASP (answer set programming)
  - model checking
  - (and more)

Combination of both:

representation *and* resolution of complex problems.

Logic and Algorithmic Decision Theory

- How does logic help representing decision making problems in a more compact, more modular, more intuitive way?
- How does logic help solving complex decision making problems?

Logic and Algorithmic Decision Theory

- How does logic help representing decision making problems in a more compact, more modular, more intuitive way?
- How does logic help solving complex decision making problems?

We'll go back and forth between logic and typical ADT problems.

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Fair Division

- $N = \{1, \ldots, n\}$ set of agents
- $O = \{o_1, \ldots, o_m\}$ indivisible objects
- allocation: maps each object to an agent
- Notation: $[o_1 o_2 | o_3 | o_4 o_5]$ is the allocation where that agent 1 receives $\{o_1 o_2\}$, 2 receives $\{o_3\}$ and 3 receives $\{o_4, o_5\}$.

"No externality" assumption:

an agent's preferences depend only on the bundle she receives

- 1 is indifferent between $[o_1 o_2 | o_3 | o_4 o_5]$ and $[o_1 o_2 | o_3 o_5 | o_4]$
- 2 is indifferent between $[o_1 o_2 | o_3 | o_4 o_5]$ and $[\varnothing | o_3 | o_1 o_2 o_4 o_5]$
- etc.

Therefore: it is sufficient to know each agent's preferences *over bundles* (as opposed to her preferences *over all allocations*).

Fair Division (here with dichotomous preferences)

- three goods: one cup of coffee, one glass of beer, one sugar cube
- three agents: J(udy), M(irek), T(orsten), with *dichotomous* preferences:
  - Judy wants a beer, or else coffee with sugar.
  - Mirek wants a beer.
  - Torsten wants a beer or a coffee.
- can they all be satisfied?
  - $b_J \vee (c_J \wedge s_J)$ where c_J means: *the coffee is allocated to Judy*
  - b_M
  - $b_T \vee c_T$
  - constraints: $b_J \rightarrow \neg b_M \wedge \neg b_T$; etc. (an object is given to at most one agent)
  - (and possibly): $b_J \vee b_M \vee b_T$ etc. (every object must be allocated)
- allocations satisfying a maximum number of agents via MAXSAT

$[b|-|c]$ $\quad + s$ to anybody (or to nobody, if allowed)
$[cs|b|-]$
$[cs|-|b]$

Dichotomous preferences for resource allocation

- $\mathcal{X} = \{o_1, \ldots, o_m\}$ set of items
- $A \subseteq \mathcal{X}$ set of acceptable bundles
- agent i partitions the set of bundles A into two sets: *acceptable* and *unacceptable* bundles
- $b_J \vee (c_J \wedge s_J)$: Judy is happy with $\{b\}$, $\{c, s\}$, $\{b, s\}$ and $\{b, c, s\}$, and unhappy with $\{c\}$, $\{s\}$, $\{b, s\}$ and $\varnothing$ [mistake]
- each set of acceptable bundles A is representable by a propositional formula φ_A
- a set of acceptable bundles A is *monotonic* if for all $X \subseteq Y$, $X \in A$ implies $Y \in A$.
- Remark A is monotonic iff φ_A is a positive formula (can be written with only $\wedge$, $\vee$, but with no $\neg$)
    - $b \wedge \neg c$ (agent allergic to the smell of coffee): nonmonotonic

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  - $b \wedge \neg c$ (agent allergic to the smell of coffee): nonmonotonic

LPNMR crowd: keep calm!

Fair Division

- Judy wants a beer, or else coffee with sugar: $b_J \vee (c_J \wedge s_J)$
- Mirek wants a beer: b_M
- Torsten wants a beer or a coffee: $b_T \vee c_T$

An allocation π is *envy-free* if every agent is at least happy with her share than with any other agent's share

- $\pi_1 = [b|-|c]$: Mirek is envious of Judy.
- $\pi_2 = [cs|b|-]$: Torsten is envious of both Judy and Mirek.
- $\pi_3 = [-|-|c]$: envy-free, but not *Pareto-efficient*: $[b|-|c]$ does at least as well as π_3 for all agents and strictly better for one (Judy).

Here: no allocation is both envy-free and Pareto-efficient

Fair Division

Preferences slightly change: Judy does not like beer anymore.

- Judy wants a coffee with sugar: $c_J \wedge s_J$
- Mirek wants a beer: b_M
- Torsten wants a beer or a coffee: $b_T \vee c_T$

- $[-|b|c]$, and also $[s|b|c]$: envy-free and Pareto-efficient

Fair Division

- Judy wants a coffee with sugar: $c_J \wedge s_J$
- Mirek wants a beer: b_M
- Torsten wants a beer or a coffee: $b_T \vee c_T$
- *EF*:

$$\begin{array}{lll} & (c_J \wedge s_J) \vee (\neg(c_M \wedge s_M) \wedge \neg(c_T \wedge s_T)) & \text{Judy not envious} \\ \wedge & b_M \vee (\neg b_J \wedge \neg b_T) & \text{Mirek not envious} \\ \wedge & (b_T \vee c_T) \vee (\neg(b_J \vee c_J) \wedge \neg(b_M \vee c_M)) & \text{Torsten not envious} \end{array}$$

- Γ: an item cannot be given to more than one person

$$c_J \rightarrow (\neg c_M \wedge \neg c_T) \wedge \ldots$$

- Pareto efficiency: satisfy a maximal subset of

$$\{c_J \wedge s_J, b_M, b_T \vee c_T\}$$

- Finding EF-PE allocations via default logic (Bouveret and L, 08):

$$\Delta = (\Gamma, D) \quad \text{where} \quad D = \left\{ \frac{:c_J \wedge s_J}{c_J \wedge s_J}, \frac{:b_M}{b_M}, \frac{:b_T \vee c_T}{b_T \vee c_T} \right\}$$

- EF-PE allocation $\leftrightarrow$ extension of Δ consistent with *EF*

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Hedonic Games

ADT-LPNMR lunch.

- Participants: Judy, Nick, Mirek, Torsten
- Judy wants to sit at a table of at least three persons.

$$(JN \wedge JT) \vee (JN \wedge JM) \vee (JT \wedge JM)$$

- Nick wants to sit at a table of exactly three persons.

$$(NJ \wedge NT \wedge \neg NM) \vee (NJ \wedge NM \wedge \neg NT) \vee (NM \wedge NT \wedge \neg NJ)$$

- Torsten wants to have lunch with Judy or Nick, but not with Mirek.

$$(TJ \vee TN) \wedge \neg TM$$

- Mirek only wants to avoid having lunch with both Judy and Nick.

$$\neg (MJ \wedge MN)$$

- Constraints: $AB \leftrightarrow BA$, $AB \wedge BC \rightarrow BC$ etc.
- What will happen?

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- What will happen?

[Judy Nick Torsten | Mirek]

everybody's happy! perfect partition

Hedonic Games

Now: dinner.

- Judy wants to sit alone, or else with Nick and Mirek.
- Nick wants to sit at a table of exactly two persons.
- Torsten does not want to have dinner alone.
- Mirek wants to have dinner with Judy.

- It is not possible to satisfy the four of them: *no perfect partition*

	Judy	Nick	Torsten	Mirek]	# happy
[Judy Nick Torsten Mirek]	+	-	+	+	3
[Judy Mirek | Nick Torsten]	-	+	+	+	3
(...)					< 3

Hedonic Games

Dinner.

- Judy wants to sit alone, or else with Nick and Mirek.
- Nick wants to sit at a table of exactly two persons.
- Torsten does not want to have dinner alone.
- Mirek wants to have dinner with Judy.

- [Judy Nick Torsten Mirek]:
  - a maximal number of players (all except Judy) are happy.
  - but not individually rational: Judy prefers to leave her coalition and eat alone.
- same thing for [Judy Mirek | Nick Torsten]
- [Torsten | Nick Mirek Judy]:
  - only two players (Judy and Mirek) are happy
  - individually rational: noone would be happier leaving their coalition and eat alone.

Hedonic Games

Dinner.

- Judy wants to sit alone, or else with Nick and Mirek.
- Nick wants to sit at a table of exactly two persons.
- Torsten does not wants to have dinner alone.
- Mirek wants to have dinner with Judy.

[Torsten | [whatever]]	$\rightarrow$ Torsten wants to join any group
[Judy Torsten | Nick | Mirek]	$\rightarrow$ Nick wants to join Mirek
[Judy | Mirek Torsten | Nick]	$\rightarrow$ Nick wants to join Judy
[Judy x | y z]	$\rightarrow$ Judy leaves and eats alone
[Nick | Judy Mirek Torsten]	$\rightarrow$ Judy leaves and eats alone
[Mirek | Judy Nick Torsten]	$\rightarrow$ Judy leaves and eats alone
[Judy | Nick Torsten | Mirek]	$\rightarrow$ Mirek wants to join Judy
[Judy | Nick Mirek Torsten]	$\rightarrow$ Mirek leaves and joins Judy
[Judy Nick Torsten Mirek]	$\rightarrow$ Judy leaves and eats alone

- no partition is Nash stable: in every partition someone prefers to leave the coalition he belongs to and join another existing coalition

Hedonic Games

Dinner.

- Judy wants to sit alone, or else with Nick and Mirek.
- Nick wants to sit at a table of exactly two persons.
- Torsten does not wants to have dinner alone.
- Mirek wants to have dinner with Judy.

[Torsten | Judy Mirek | Nick]	→ Torsten wants to join Nick; Nick: yes!
[Judy Torsten | Nick | Mirek]	→ Nick wants to join Mirek; Mirek: yes!
[Judy | Mirek Torsten | Nick]	→ Nick wants to join Judy; Judy: sorry, no
	→ Mirek wants to join Judy; Judy: sorry, no
	→ noone else wants to deviate.

- [Judy | Mirek Torsten | Nick] is individually stable: noone prefers joining another coalition without making a member of this coalition less happy.
- Logical characterization of solution concepts in dichotomous hedonic games in (Aziz, Harrenstein, L and Wooldridge, 14)
- Related: *group activity selection*, cf. talk by Andreas Darmann on Monday

Preference structures

In the latter two examples, preferences are *dichotomous*. More generally:

Ordinal preferences

Preference relation on $\mathcal{X}$: **reflexive and transitive relation** $\succeq$

$x \succeq y$		x is at least as good as y
$x \succ y$	$\Leftrightarrow$	$x \succeq y$ and not $y \succeq x$
		x is preferred to y (**strict preference**)
$x \sim y$	$\Leftrightarrow$	$x \succeq y$ and $y \succeq x$
		x and y are equally preferred (**indifference**)

$\succeq$ is often assumed to be complete (no incomparabilities)

Cardinal preferences

- Utility function $u : \mathcal{X} \to \mathbb{R}$
- More generally $u : \mathcal{X} \to V$ ordered scale; example:
  $V = \{unacceptable, bad, medium, good, excellent\}$

Preference structures

From cardinal preferences to ordinal preferences:

$$x \succeq_u y \Leftrightarrow u(x) \geq u(y)$$

Dichotomous preferences are back

- $A \subseteq \mathcal{X}$ set of acceptable bundles
- dichotomous preferences are cardinal preferences:

$$V = \{0, 1\}; \;\; u(S) = 1 \Leftrightarrow S \in A.$$

- dichotomous preferences are also ordinal preferences:

$$S \succeq S' \Leftrightarrow (S \in A) \text{ or } (S' \notin A).$$

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Weighted Goals

- L_{PS} propositional language built up from usual connectives and set of propositional symbols PS.
- G = a set of pairs $\langle \varphi_i, w_i \rangle$ where
  - φ_i is a propositional formula;
  - w_i is a real number
- for every truth assignment (interpretation) $x \in 2^{PS}$,

$$u_G(x) = \sum \{ w_i \mid \langle \varphi_i, w_i \rangle \in G \text{ and } x \models \varphi_i \}$$

Combinatorial Auctions

- $\mathcal{O} = \{o_1, \ldots, o_m\}$ set of objects
- for each agent i, $V_i : 2^{\mathcal{O}} \rightarrow \mathbb{N}$ where $V_i(X)$ is the maximum price that i is ready to pay for the set of objects X.
- V_i is *additive* if $V_i(X) = \sum_{o \in X} V_i(o)$ for all X.
- if V_i additive for all i: then sell each object to its highest bidder
- but V_i is generally *non-additive* :
  - {left shoe}: 10 €; {right shoe}: 10 €; {left shoe, right shoe}: 50 €
  - {lemonade}: 2 €; {beer}: 3 €; {lemonade, beer}: 4 €
- optimal allocation π^*: maximizes the seller's revenue

$$\sum_{i=1}^{n} V_i(\pi(i))$$

  where $\pi(i)$ is the set of objects allocated to agent i
- How can bidders express their functions V_i?
- How can the seller determine π^*?

Combinatorial Auctions through Weighted Goals

- adapted from (Boutilier and Hoos, 2001)
- items: 3 chopsticks c_1, c_2, c_3; one fork f, one knife k
- $2chopsticks = (c_1 \wedge c_2) \vee (c_1 \wedge c_3) \vee (c_2 \wedge c_3)$
- Judy:

$$\{(2chopsticks \vee fork, 5), (fork \wedge knife, 1), (2chopsticks, 3)\}$$

- Mirek:

$$\{(2chopsticks, 2), (fork, 4), (fork \wedge knife, 4), 1)\}$$

- Torsten:

$$\{(2chopsticks \vee fork, 6), (fork \wedge (c_1 \vee c_2 \vee c_3), 1)\}$$

- Who gets what?

	$2c$	f	$f+k$	$f+c$
Judy	8	5	6	5
Mirek	2	4	8	4
Torsten	6	6	6	7

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- Judy:

$$\{(2chopsticks \vee fork, 5), (fork \wedge knife, 1), (2chopsticks, 3)\}$$

- Mirek:

$$\{(2chopsticks, 2), (fork, 4), (fork \wedge knife, 4), 1)\}$$

- Torsten:

$$\{(2chopsticks \vee fork, 6), (fork \wedge (c_1 \vee c_2 \vee c_3), 1)\}$$

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Prioritized goals

- starts with (Brewka, 89)
- $G = \langle G_1, \ldots, G_q \rangle$
- G_i set of goals φ_i^j of priority i – each being a propositional formula
- G_1= set of highest priority goals, then G_2 etc.
- maximize the number (or the set) of goals satisfied, starting from the most important priority levels
- particular case: *conditionally lexicographic preferences* (cf. talk by Xudong Liu on Monday)
- two semantics (coinciding if each G_i is a singleton):
- leximin $x \succ y$ if there is a $k \leq q$ such that
  - $|\{\varphi \in G_i, x \models \varphi\}| = |\{\varphi \in G_i, y \models \varphi\}|$;
  - for each $i < k$: $|\{\varphi \in G_i, x \models \varphi\}| = |\{\varphi \in G_i, y \models \varphi\}|$.
- discrimin $x \succ y$ if there is a $k \leq q$ such that
  - $\{\varphi \in G_i, x \models \varphi\} \supset \{\varphi \in G_i, y \models \varphi\}$;
  - for each $i < k$: $\{\varphi \in G_i, x \models \varphi\} = \{\varphi \in \varphi, y \models \varphi\}$.

Multiple Referenda

Lexingtonians called to urns:

- should we build a new university campus or not? (c or $\neg c$)
- should we build a tram or not? (t or $\neg t$)
- should we build a new horse race field or not? (h or $\neg h$)

- Judy's prioritized goals: $G_1 = \{\neg(c \wedge t \wedge h)\}$, $G_2 = \{c\}$, $G_3 = \{t\}$
- Judy's induced preference relation:

$$
\begin{array}{c}
ct\overline{h} \\
\downarrow \\
c\overline{t}h \sim c\overline{t}\overline{h} \\
\downarrow \\
\overline{c}th \sim \overline{c}t\overline{h} \\
\downarrow \\
\overline{c}\overline{t}h \sim \overline{c}\overline{t}\overline{h} \\
\downarrow \\
cth
\end{array}
$$

Multiple Referenda

- Judy: $G_1 = \{\neg(c \wedge t \wedge h)\}$, $G_2 = \{c\}$, $G_3 = \{t\}$

  $ct\bar{h} \succ \ldots$

- Mirek: $G_1 = \{\neg(c \wedge t \wedge h)\}$, $G_2 = \{t\}$, $G_3 = \{h\}$

  $\bar{c}th \succ \ldots$

- Nick: $G_1 = \{\neg(c \wedge t \wedge h)\}$, $G_2 = \{h\}$, $G_3 = \{c\}$

  $c\bar{t}h \succ \ldots$

If we vote separately on each issue, the following outcome may occur:

- Judy and Nick vote for c, Mirek against;
- Judy and Mirek vote for t, Nick against;
- Mirek and Nick vote for h, Judy against
- Outcome: cth – is it good?

Multiple Referenda

- Judy: $G_1 = \{\neg(c \wedge t \wedge h)\}$, $G_2 = \{c\}$, $G_3 = \{t\}$

  $ct\overline{h} \succ \ldots$

- Mirek: $G_1 = \{\neg(c \wedge t \wedge h)\}$, $G_2 = \{t\}$, $G_3 = \{h\}$

  $\overline{c}th \succ \ldots$

- Nick: $G_1 = \{\neg(c \wedge t \wedge h)\}$, $G_2 = \{h\}$, $G_3 = \{c\}$

  $c\overline{t}h \succ \ldots$

If we vote separately on each issue, the following outcome may occur:

- Judy and Nick vote for c, Mirek against;
- Judy and Mirek vote for t, Nick against;
- Mirek and Nick vote for h, Judy against
- Outcome: cth – is it good?

Need for more sophisticated methods!

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Preference logics

A I prefer to go to Chicago tomorrow by bus than by plane

Preference logics

A I prefer to go to Chicago tomorrow by bus than by plane

Can we infer from A the following?

B I prefer to go to Chicago tomorrow by bus and have a beer tonight than by plane and have a beer tonight.

Preference logics

A I prefer to go to Chicago tomorrow by bus than by plane

Can we infer from A the following?

B I prefer to go to Chicago tomorrow by bus and have a beer tonight than by plane and have a beer tonight.

C I prefer to go to Chicago tomorrow by bus with a strong toothache than by plane after seeing a good dentist.

Preference logics

- A I prefer to go to Chicago tomorrow by bus than by plane

Can we infer from A the following?

- B I prefer to go to Chicago tomorrow by bus and have a beer tonight than by plane and have a beer tonight.
- C I prefer to go to Chicago tomorrow by bus with a strong toothache than by plane after seeing a good dentist.
- D I prefer to go to Chicago tomorrow by bus (7 hours) with a strong toothache than by plane with a strong toothache.

Preference logics

Classic preference logic (von Wright, 1963)

- formulas built up from preference statements $\alpha \rhd \beta$
- $\alpha \wedge \neg\beta$-worlds preferred to $\beta \wedge \neg\alpha$-worlds, *ceteris paribus*
- here *ceteris paribus* means that all variables not appearing in α or β must be interpreted identically

- $\mathit{bus} \rhd \mathit{plane}$:
  - implies $(\mathit{bus}, \mathit{beer}, \neg\mathit{toothache}) \succ (\mathit{plane}, \mathit{beer}, \neg\mathit{toothache})$
  - $(\mathit{bus}, \mathit{beer}, \mathit{toothache})$ and $(\mathit{plane}, \mathit{beer}, \neg\mathit{toothache})$ incomparable
  - $(\mathit{bus}, \mathit{beer}, \mathit{toothache})$ and $(\mathit{bus}, \neg\mathit{beer}, \neg\mathit{toothache})$ incomparable
- $\mathit{toothache} \wedge \mathit{plane} \rhd \mathit{toothache} \wedge \mathit{bus}$ [shorthand $\mathit{toothache} : \mathit{plane} \rhd \mathit{bus}$]
  - $(\mathit{bus}, \mathit{beer}, \neg\mathit{toothache}) \succ (\mathit{plane}, \mathit{beer}, \neg\mathit{toothache})$
  - $(\mathit{bus}, \mathit{beer}, \mathit{toothache})$ and $(\mathit{plane}, \mathit{beer}, \neg\mathit{toothache})$ still incomparable
  - $(\mathit{bus}, \mathit{beer}, \mathit{toothache})$ and $(\mathit{bus}, \neg\mathit{beer}, \neg\mathit{toothache})$ still incomparable

Preference logics

- Modern preference logics: Hansson (2001), van Benthem, Roy and Girard. (2009), Bienvenu, L and Wilson (2010), etc.
- PL formulas are Boolean combinations of preference statements of the form

$$\alpha \rhd \beta \;||\; F$$

  α, β propositional formulas, F a set of propositional formulas
- α preferred to β when F is held constant; other formulas can vary
- formally: $\succ$ satisfies $(\alpha \rhd \beta \;||\; F)$ if $\omega \succ \omega'$ holds for all ω, ω' such that
  - $\omega \models \alpha$
  - $\omega' \models \beta$
  - forall $\varphi \in F$: $\omega \models \varphi$ if and only if $\omega' \models \varphi$.
- $\neg toothache \rhd toothache \;||\; \varnothing$:
  - $(bus, \neg beer, \neg toothache) \succ (plane, beer, toothache)$
- $beer \rhd \neg beer \;||\; \{bus, plane, toothache\}$
  shorthand: $beer \rhd \neg beer \;||\; CP$, where CP = *ceteris paribus*

Preference logics

Many existing formalisms can be seen as fragments of PL:

- von Wright's preference logic
- conditional preference (CP) networks (Boutilier *et al.*, 2003)
- extensions of CP-nets (TCP-nets, etc.)
- conditional importance networks (Bouveret, Endriss and L, 2009)
- prioritized goal bases (Brewka, 89)

One talk, two plans

- Common appetizer

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Multiattribute decision making

Toby travels (except when he cannot). He is considering buying

- an outgoing flight (o),
- a return flight (r),
- a hotel night (h),
- a book (b).

His preferences:

- better both tickets than none, and better none than just one; preferences about tickets override everything else

$$(o \wedge r) \rhd (\neg o \wedge \neg r) \rhd (o \leftrightarrow \neg r) \mid\mid \varnothing$$

- he wants a hotel night if and only if he buys a return flight ticket

$$o \wedge r : h \rhd \neg h \mid\mid \{o \leftrightarrow r\}$$

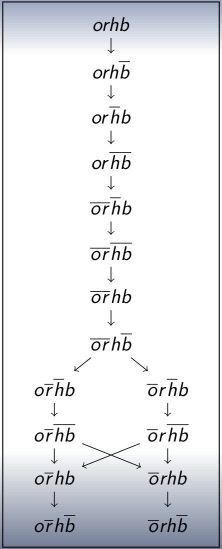
$$\neg(o \wedge r) : \neg h \rhd h \mid\mid \{o \leftrightarrow r\}$$

- he wants to buy the book, *ceteris paribus*

$$b \rhd \neg b \mid\mid \{o, r, h\}$$

Multiattribute decision making

- $(o \wedge r) \rhd (\neg o \wedge \neg r) \rhd (o \leftrightarrow \neg r) \; || \; \varnothing$
  - $or \times \times \succ \overline{or} \times \times$
  - $\overline{or} \times \times \succ o\overline{r} \times \times$
  - $\overline{or} \times \times \succ \overline{o}r \times \times$
- $o \wedge r : h \rhd \neg h \; || \; \{o \leftrightarrow r\}$
  - $orh\times \succ or\overline{h}\times$
- $\neg(o \wedge r) : \neg h \rhd h \; || \{o \leftrightarrow r\}$
  - $\overline{o}r\overline{h}\times \succ \overline{o}rh\times$
  - $\overline{o}r\overline{h}\times \succ o\overline{r}h\times$
  - $o\overline{r}\overline{h}\times \succ o\overline{r}h\times$
  - $o\overline{r}\overline{h}\times \succ \overline{o}rh\times$
- $b \rhd \neg b \; || \; \{o, r, h\}$
  - $orhb \succ orh\overline{b}$
  - $or\overline{h}b \succ or\overline{hb}$ etc.



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Committee Elections

- two seats to fill for the department managing committee
- candidates: A, B, C, D, E

	woman	man
group 1	A,E	B
group 2	C	D

- preferences of voter 1:
  - 1M+1W $\triangleright$ 2M $\sim$ 2W $\|$ $\varnothing$
    where: $1M{+}1W = (A \wedge B \wedge \neg C \wedge \neg D \wedge \neg E) \vee (E \wedge B \wedge \neg A \wedge \neg C \wedge \neg D) \vee (\ldots)$
    gender equilibrium more important than everything else
  - 1G1+1G2 $\triangleright$ 2G2 $\triangleright$ 2G1 $\|$ {1M1W, 2M, 2W}
    group equilibrium most important thing after gender equilibrium
  - $A \triangleright B \triangleright C \triangleright D \triangleright E \,\|\,$ {1M1W, 2M, 2W, 1G1+1G2,2G1,2G2} (*ceteris paribus*)

Committee Elections

	woman	man
group 1	A,E	B
group 2	C	D

- 1M+1W $\rhd$ 2M $\sim$ 2W $||$ $\varnothing$
- 1G1+1G2 $\rhd$ 2G2 $\rhd$ 2G1 $||$ {1M1W, 2M, 2W}
- $A \rhd B \rhd C \rhd D \rhd E$ $||$ {1M1W, 2M, 2W, 1G1+1G2,2G1,2G2}

Induced preference relation for voter 1:

AD ↘
DE → CD → AB → BE → AC → BD → CE → AE
BC ↗

Voter 1's preferred committee is *AD* or *BC* – we don't have enough information to know which one.

Committee Elections

- Voter 1's preferred committee: *AD* or *BC*
- Voter 2's preferred committee: *AE* or *BE*
- Voter 3's preferred committee: *BD*

Standard rule for multiwinner approval voting (also called 'minisum'):

- each voter votes for her preferred committee
- the (here: two) candidates that appear most often on the votes are elected
- tie-breaking priority = age: $D > E > A > B > C$

	$1 : AD$	$1 : BC$
$2 : AE$	$12021 \mapsto BD$	$03111 \mapsto BD$
$2 : BE$	$21021 \mapsto AD$	$12111 \mapsto BD$

- *D* is a necessary winner
- *A* and *B* (and of course *D*) are possible winners

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Nonmonotonic Preferences

Should we have a department meeting on Monday?

- yes, I prefer to have the department meeting this Monday
- if there's a train strike, I'd prefer to cancel the department meeting
- if Barack Obama intends to visit the department on Monday, then yes, I'd prefer to have the meeting in any case (even if there is a strike)

◇ normal situation: no strike, no Obama

◇ exceptional situation: strike, no Obama

◇ even more exceptional situation: Obama

Nonmonotonic Preferences

preference order		normality order	
$m\bar{s}o$	most preferred		
$\downarrow$			
mso		$\times\overline{so}$	normal
$\downarrow$		$\downarrow$	
$m\overline{so}$		$\times s\bar{o}$	exceptional
$\downarrow$		$\downarrow$	
$\overline{m}\times\times$		$\times\times o$	super-exceptional
$\downarrow$			
$ms\bar{o}$	least preferred		

Nonmonotonic Preferences

preference order		normality order	
$m\bar{s}o$	most preferred		
$\downarrow$		$m\bar{s}\bar{o}, \bar{m}\bar{s}\bar{o}$	normal
mso		$\downarrow$	
$\downarrow$		$ms\bar{o}, \bar{m}s\bar{o}$	exceptional
$m\bar{s}\bar{o}$		$\downarrow$	
$\downarrow$			
$\bar{m}so, \bar{m}\bar{s}o, \bar{m}s\bar{o}, \bar{m}\bar{s}\bar{o}$		$mso, \bar{m}so,$ $m\bar{s}o, \bar{m}\bar{s}o$	super-exceptional
$\downarrow$			
$ms\bar{o}$	least preferred		

$\varphi : m \succ_P \neg m$ if typical $\varphi \wedge m$-worlds preferred to typical $\varphi \wedge \neg m$-worlds

most normal m-world	$m\bar{s}\bar{o}$	
	$\downarrow_P$	$m \succ_P \neg m$
most normal $\neg m$-world	$\bar{m}\bar{s}\bar{o}$	

Nonmonotonic Preferences

preference order	
$m\bar{s}o$	most preferred
$\downarrow$	
mso	
$\downarrow$	
$m\bar{s}\bar{o}$	
$\downarrow$	
$\bar{m}so, \bar{m}\bar{s}o, \bar{m}s\bar{o}, \bar{m}\bar{s}\bar{o}$	
$\downarrow$	
$ms\bar{o}$	least preferred

normality order	
$m\bar{s}\bar{o}, \bar{m}\bar{s}\bar{o}$	normal
$\downarrow$	
$ms\bar{o}, \bar{m}s\bar{o}$	exceptional
$\downarrow$	
$mso, \bar{m}so, m\bar{s}o, \bar{m}\bar{s}o$	super-exceptional

$\varphi : m \succ_P \neg m$ if typical $\varphi \wedge m$-worlds preferred to typical $\varphi \wedge \neg m$-worlds

most normal $s \wedge m$-world $ms\bar{o}$

$\uparrow_P$ $\quad s : \neg m \succ_P m$

most normal $s \wedge \neg m$-world $\bar{m}s\bar{o}$

Nonmonotonic Preferences

preference order		normality order	
$m\bar{s}o$	most preferred		
$\downarrow$		$m\bar{s}\bar{o}, \bar{m}\bar{s}\bar{o}$	normal
mso		$\downarrow$	
$\downarrow$		$ms\bar{o}, \bar{m}s\bar{o}$	exceptional
$m\bar{s}\bar{o}$		$\downarrow$	
$\downarrow$		$mso, \bar{m}so,$ $m\bar{s}o, \bar{m}\bar{s}o$	super-exceptional
$\bar{m}so, \bar{m}\bar{s}o, \bar{m}s\bar{o}, \bar{m}\bar{s}\bar{o}$			
$\downarrow$			
$ms\bar{o}$	least preferred		

$\varphi : m \succ_P \neg m$ if typical $\varphi \wedge m$-worlds preferred to typical $\varphi \wedge \neg m$-worlds

most normal $o \wedge s \wedge m$-world $\quad mso$

$\downarrow_P \qquad o \wedge s : m \succ_P \neg m$

most normal $o \wedge s \wedge \neg m$-world $\quad \bar{m}so$

Nonmonotonic Preferences

Another example:

- I don't want to have the meeting on Monday
- but if we do have it on Monday, then I want to have my lecture on Monday afternoon.

  (cf. contrary-to-duties obligations in deontic logics)

Nonmonotonic Preferences

Yet another example: Lexingtonian are called to urns again

- should we build a new university campus? u or $\overline{u}$
- should we build a tram? t or $\overline{t}$
- should we build a new horse race field? h or $\overline{h}$

Judy's preferences:

- $u \succ \overline{u}$
- $t \succ \overline{t}$
- $h \succ \overline{h}$
- but $u \wedge t : \overline{h} \succ h$
- Judy believes that $u \wedge t$ is very unlikely.
- Therefore she intends to vote for *yes* for u, for t and for h
- But now, the Lexington Post publishes a poll: it's likely that u an t will get a slight majority of yes!
- Judy now votes *yes* for u, yes for t and *no* for h

Defeasible Beliefs vs. Defeasible Preferences

- W set of worlds
- $\succeq_N$ *normality ordering* (complete weak order on W)
- $\succeq_P$: *preference ordering* (complete weak order on W)

normality

- $N(\beta|\alpha)$: if α then normally, β
- $N(\beta|\alpha)$ is satisfied if the most normal α-worlds satisfy β
- formally: if $Max(\succeq_N, Mod(\alpha)) \subseteq Mod(\beta)$

preference things are less obvious, for two reasons:

1. there is no standard way of lifting preferences from worlds to sets of worlds.
2. in the presence of uncertainty or normality, preferences can hardly be interpreted from $\succeq_P$ alone ($\succeq_N$ counts!).

Defeasible Beliefs vs. Defeasible Preferences

Step 1: lifting preferences from worlds to sets of worlds

- $\succeq_P$ complete weak orders on W
- we want to lift $\succeq_P$ from W to 2^W
- W_1, W_2 nonempty sets of worlds
- $W_1 \gg W_2$ if ...

strong lifting every world in W_1 is preferred to every world in W_2.

optimistic lifting the 'best' (most preferred) worlds in W_1 are preferred to the best worlds in W_2.

pessimistic lifting the worst worlds in W_1 are preferred to the worst worlds in W_2.

ceteris paribus lifting the worlds in W_1 are preferred to the worlds in W_2, *ceteris paribus*

(etc.)

Defeasible Beliefs vs. Defeasible Preferences

Step 2: interpreting preference in the presence of normality

- When an agent states a preference for φ he not only expresses preferences between worlds but also to *implicit uncertainty/normality*.
- At least two meaningful definitions:

Boutilier, 94 *among the most normal α-worlds, the β-worlds are preferred to the $\neg\beta$-worlds*

L, van der Torre and Weydert, 03 *the most normal $\alpha \wedge \beta$-worlds are preferred to the most normal $\alpha \wedge \neg\beta$-worlds*

Nonmonotonic Doodle

- str: train strike; sc: seminar cancelled; h: hurricane
- $N(sc|h)$: normally, seminar cancelled when hurricane.
- train strikes and hurricanes known the day before
- seminar cancellations known two days before, except when hurricane

	Monday	Tuesday	Wednesday
Judy	Y str: N	N sc: Y	N
Mirek	Y	N sc: Y; h: N	Y
Nick	N	Y	Y
Toby	Y	Y	Y
Torsten	Y str: N	Y h: N	N
best date	?	?	?

Other

Because of lack of time I did not talk about

- Description logics for multi-attribute decision making (cf. talk by Erman Acar on Monday)
- Judgment aggregation (cf. talk by Ann-Kathrin Selker this morning)
- Boolean games
- (and yet other things)

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Logic programming for ADT

The ADT family

planning multiple referenda cooperative games
hedonic games resource allocation committee elections
multiattribute decision making noncooperative games (...)

- need a modular, compact and declarative representation of the problem
- high complexity (often above NP)

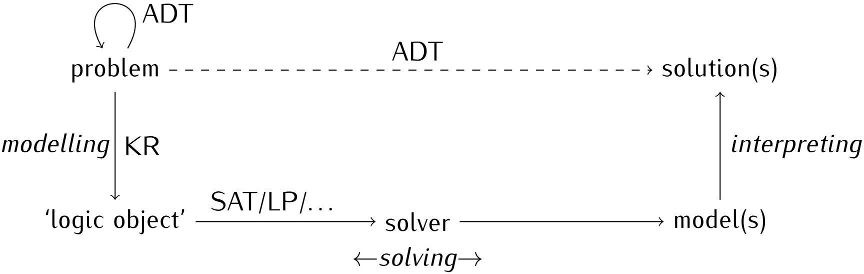
The ASP family

AnsProlog ASPeRIX ASSAT Clasp clingo Cmodels coala DLV DLV-Complex
GnT gringo iclingo libdlvhexbase6-dev lparse NoMoRe++ Platypus
Pbmodels Potassco relsat runlim Smodels Smodels-cc Sup-lp (...)

- declarative problem representation
- generic resolution tool for hard combinatorial problems
- built-in preference handling — e.g., *Asprin*

ASP for ADT

[Warning: plagiarizing]



Topcasso

Logic programming for ADT

Generic use of LPNMR for ADT: topic of three talks at ADT-LPNMR 15

- Andreas Pfandler, Democratix, A Declarative Approach to Winner Determination, on Tuesday
- Torsten Schaub, Implementing Preferences with *asprin*, on Tuesday
- myself, Algorithmic Decision Theory Meets Logic, right now (warning: this is an auto-referential talk)

Science-fiction: ADT-LPNMR 2017

Program:

- Winner Determination and Manipulation in Minimax Committee Elections via Infinitary Equilibrium Logic and Strong Equivalence

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Complexity of Bayesian Sequential Manipulation and Control in OWA-Based Extensions of Uniform Weighted Incomplete Resource Allocation: Approximation and Super-strong Equilibria

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  with *asprin*