Stable Models for Temporal Theories

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Joint work with KR group at

UNIVERSIDADE DA CORUÑA

Felicidad Aguado

Martín Diéguez

Gilberto Pérez

Concepción Vidal

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and collaborators from other universities:

- David Pearce and Laura Bozzelli (Univ. Pol. Madrid, Spain),
- Stephane Demri (ENS de Cachan, France),
- Philippe Balbiani and Luis Fariñas (IRIT Toulouse, France)

Initial motivation

Knowledge
Representation

Formalizing dynamic domains was part of KR origins

- Actions and Change: temporal domains in first-order logic
  - Situation Calculus
  - Event Calculus
  - Features and Fluents

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- Actions and Change: temporal domains in first-order logic
  - Situation Calculus
  - Event Calculus
  - Features and Fluents
- Representational problems: frame, Yale Shooting, . . .
  How to deal with defaults like inertia?

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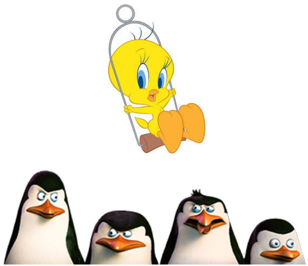

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- Well, it is not flying . . .

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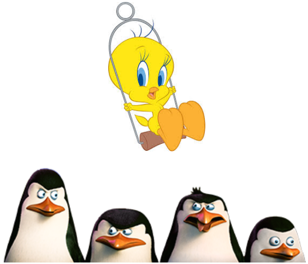

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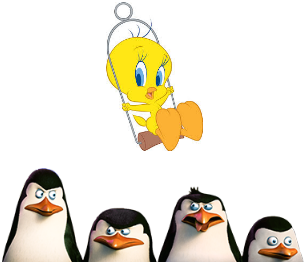

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Late 80's, strong connection between

LP	-	**NMR**
Logic Programming		Non-Monotonic Reasoning

Example of correspondence:

LP - NMR

$$\begin{array}{lcl} p & \leftarrow & not\ q \\ r & \leftarrow & p, not\ s \end{array} \quad = \quad \frac{:\ \neg q}{p} \quad \frac{p\ :\ \neg s}{r}$$

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- Why not using logic programs for action and change?
- [Gelfond & Lifschitz, JLP 93]
  Representing Action and Change by Logic Programs
  Established a new methodology giving rise to ...

Transition systems in ASP

Transition systems in Answer Set Programming (ASP)

Some nice features

- ✓ Elaboration tolerance: small changes in the problem ⇒ small changes in representation
- ✓ Simple solution to frame, ramification and qualification problems

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- ✓ Incremental ASP exploits time index to reuse grounding/solving

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But not thought for temporal reasoning

- ✗ Planning by iterative deepening with finite path length:
  we cannot prove non-existence of plan
- ✗ Reactive systems out of the scope:
  e.g. a network server must keep on running (potentially) forever

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- ✗ Existing formal methods for transition systems: outside ASP

Transition systems in ASP

Example

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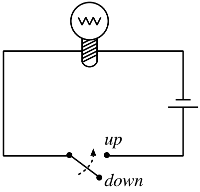


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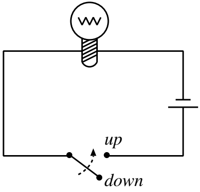


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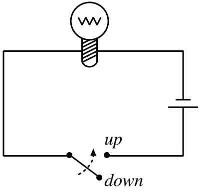


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- Is there a reachable state with *up* and *down* false?
- Once *up* becomes true, does it remain so forever?
- The switch cannot be closed infinitely often without eventually damaging the lamp

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  algorithms, computability, complexity, formal verification
- But, initially, not so much in Actions and Change
  [McCarthy97] "*Modality, si! Modal logic, no!*"
- Nowadays, temporal logics used in KR or planning,
  but difficult combination with NMR

Modal Temporal Logic

A simple and well-known case

Linear-time Temporal Logic (LTL)

$\square$ (forever), $\Diamond$ (eventually), $\bigcirc$ (next), $\mathcal{U}$ (until)

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- ✓ Model checking and verification of reactive systems
- ✓ Many uses in AI: planning, ontologies, multi-agent systems, . . .

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- ✗ NMR attempts for LTL: limited syntax, only for queries, control rules, etc. Not really embodied in LTL

Our proposal

Temporal Equilibrium Logic (TEL) [C_&Pérez 07]

TEL = ASP + LTL

- ASP: logical characterisation Equilibrium Logic [Pearce 96]
- LTL: We add temporal operators $\square$, $\Diamond$, $\bigcirc$, $\mathcal{U}$, $\mathcal{R}$.

Result: Temporal Stable Models for any arbitrary LTL theory.

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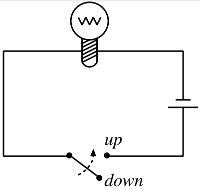


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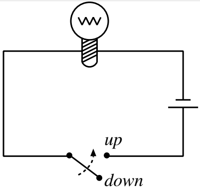

$up \vee down$			Initially
$\Box(\bigcirc up$	$\leftarrow$	$up \wedge \neg \bigcirc down)$	Inertia
$\Box(\bigcirc down$	$\leftarrow$	$down \wedge \neg \bigcirc up)$	Inertia
$\Box(up \vee \neg up)$			Choice

Idea: LTL syntax, but keeping ASP semantics

Equilibrium Logic

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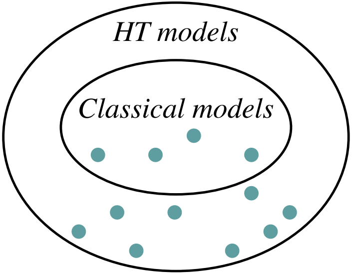

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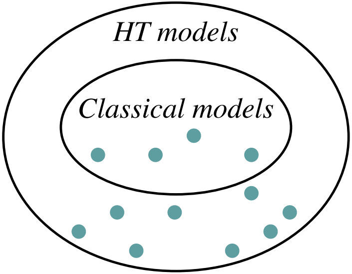

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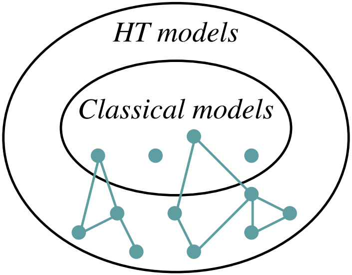

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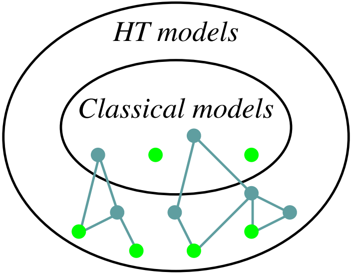

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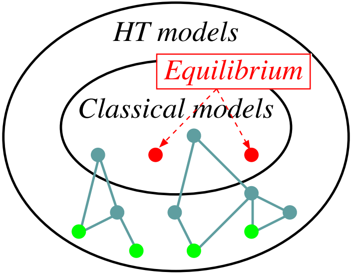

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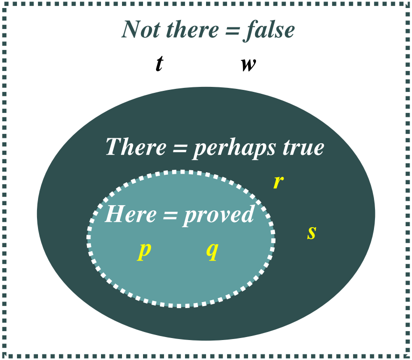
- Example: $H = \{p, q\}, T = \{p, q, r, s\}$. Intuition:



- When $H = T$ we have a classical model.

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Satisfaction of formulas

$\langle H, T \rangle \models \varphi \quad \Leftrightarrow \quad$ "φ is proved"

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- $\langle H, T \rangle \models \varphi \to \psi$ if both
  - $T \models \varphi \to \psi$ classically
  - $\langle H, T \rangle \models \varphi$ implies $\langle H, T \rangle \models \psi$

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- Negation $\neg F$ is defined as $F \rightarrow \bot$
- $\langle H, T\rangle \models \varphi$ implies $T \models \varphi$ (proved implies potentially true)
- Relation to Gelfond & Lifschitz's reduct:
  $\langle H, T\rangle \models P$ iff $H \models P^T$ classically

Equilibrium models

Definition (Equilibrium/stable model)

A model $\langle T, T\rangle$ of Γ is an equilibrium model iff

$$\text{there is no } H \subset T \text{ such that } \langle H, T\rangle \models \Gamma.$$

When this holds, T is called a stable model.

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In other words, all assumptions T are eventually proved H

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- Two theories Γ_1, Γ_2 are strongly equivalent if $\Gamma_1 \cup \Gamma$ and $\Gamma_2 \cup \Gamma$ have the same equilibrium models for any Γ.

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  Strong equivalence of equilibrium theories = HT equivalence [Lifschitz, Pearce, Valverde 01].

- Captures all LP extensions with propositional connectives (also first-order [Pearce & Valverde 04]).

- Moreover, covers arbitrary formulas, in a very reasonable way:

$$\text{intuitionistic} \subset \text{HT} \subset \text{classical}$$

(Linear) Temporal Equilibrium Logic

- Syntax = propositional plus
  - $\Box\varphi$ = "forever" φ
  - $\Diamond\varphi$ = "eventually" φ
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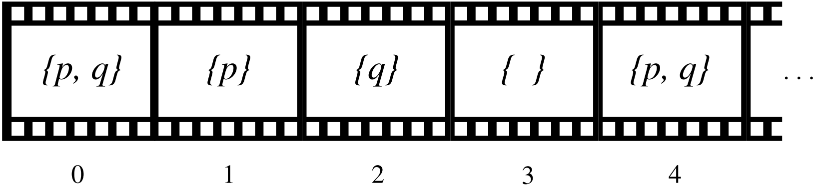
- As we had with Equilibrium Logic:
  1. A monotonic underlying logic: Temporal Here-and-There (THT)
  2. An ordering among models. Select minimal models.

- In standard LTL, interpretations are ∞ sequences of sets of atoms



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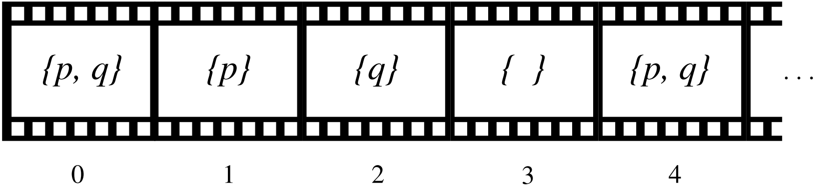
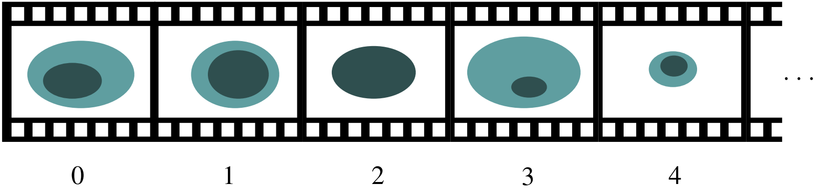


- In THT we will have ∞ sequences of HT interpretations



Sequences

- We define an ordering among sequences $\mathbf{H} \leq \mathbf{T}$ when

$$
\begin{array}{ccccccccccc}
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\cup| & & \cup| & & \cup| & & & & \cup| & & \\
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Definition (THT-interpretation)

is a pair of sequences of sets of atoms $\langle \mathbf{H}, \mathbf{T} \rangle$ with $\mathbf{H} \leq \mathbf{T}$. □

Sequences

- We define an ordering among sequences $\mathbf{H} < \mathbf{T}$ when

$$\begin{array}{ccccccccccc}
T_0 & \longrightarrow & T_1 & \longrightarrow & T_2 & \longrightarrow & \ldots & \longrightarrow & T_i & \longrightarrow & \ldots \\
\cup\!| & & \cup\!| & & \cup & & & & \cup\!| & & \\
H_0 & \longrightarrow & H_1 & \longrightarrow & H_2 & \longrightarrow & \ldots & \longrightarrow & H_i & \longrightarrow & \ldots
\end{array}$$

Definition (THT-interpretation)

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Temporal Here-and-There (THT)

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- An interpretation $M = \langle \mathbf{H}, \mathbf{T} \rangle$ satisfies α at situation i, written $M, i \models \alpha$

α	$M, i \models \alpha$ when ...
an atom p	$p \in H_0$
$\wedge, \vee$	as usual
$\varphi \rightarrow \psi$	$\mathbf{T}, i \models \varphi \rightarrow \psi$ in LTL and $\langle \mathbf{H}, \mathbf{T} \rangle, i \models \varphi$ implies $\langle \mathbf{H}, \mathbf{T} \rangle, i \models \psi$

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α	$M, i \models \alpha$ when ...
$\bigcirc\varphi$	$(M, i{+}1) \models \varphi$
$\Box\varphi$	$\forall j \geq i, \quad M, j \models \varphi$
$\Diamond\varphi$	$\exists j \geq i, \quad M, j \models \varphi$
$\varphi \, \mathcal{U} \, \psi$	$\exists j \geq i, \quad M, j \models \psi$ and $\forall k$ s.t. $i \leq k < j, \quad M, k \models \varphi$
$\varphi \, \mathcal{R} \, \psi$	$\forall j \geq i, \quad M, j \models \psi$ or $\exists k, i \leq k < j, \quad M, k \models \varphi$

- M is a model of a theory Γ when $M, 0 \models \alpha$ for all $\alpha \in \Gamma$

(Linear) Temporal Equilibrium Logic

- $\bigcirc\varphi$

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- $\Box\varphi$

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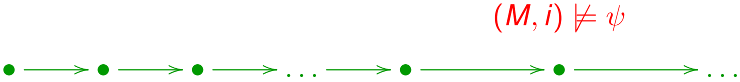

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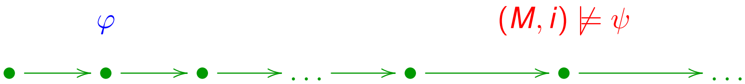

Temporal Here-and-There (THT)

- Some valid THT formulas:

$$
\begin{array}{rcl}
\Diamond\varphi & \leftrightarrow & \top\,\mathcal{U}\,\varphi \\
\Box\varphi & \leftrightarrow & \bot\,\mathcal{R}\,\varphi \\
\bigcirc(\varphi \otimes \psi) & \leftrightarrow & \bigcirc\varphi \otimes \bigcirc\psi \\
\varphi\,\mathcal{U}\,\psi & \leftrightarrow & \psi \vee (\varphi \wedge \bigcirc(\varphi\,\mathcal{U}\,\psi)) \\
\varphi\,\mathcal{R}\,\psi & \leftrightarrow & \psi \wedge (\varphi \vee \bigcirc(\varphi\,\mathcal{R}\,\psi)) \\
\neg(\varphi\,\mathcal{U}\,\psi) & \leftrightarrow & \neg\varphi\,\mathcal{R}\,\neg\psi \\
\bigcirc\neg\varphi & \leftrightarrow & \neg\bigcirc\varphi \\
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For $\otimes = \wedge, \vee, \rightarrow, \mathcal{U}, \mathcal{R}$.

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For $\otimes = \wedge, \vee, \rightarrow, \mathcal{U}, \mathcal{R}$.

- Axiomatization of THT: ongoing work [Balbiani & Diéguez 15]

Temporal Equilibrium Models

Definition (Temporal Equilibrium Model)

of a theory Γ is a model $M = \langle \mathbf{T}, \mathbf{T} \rangle$ of Γ such that there is no $\mathbf{H} < \mathbf{T}$ satisfying $\langle \mathbf{H}, \mathbf{T} \rangle, 0 \models \Gamma$. □

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- Temporal Equilibrium Logic (TEL) is the logic induced by temporal equilibrium models.

Definition (Temporal Stable Model)

$\mathbf{T}$ is a temporal stable model of a theory Γ iff $\langle \mathbf{T}, \mathbf{T} \rangle$ is a temporal equilibrium model of Γ. □

Some examples

- Example 1: TEL models of $\Box(\neg p \rightarrow \bigcirc p)$. It's like an infinite program:

$$\begin{array}{rcl} \neg p & \rightarrow & \bigcirc p \\ \neg \bigcirc p & \rightarrow & \bigcirc^2 p \\ \neg \bigcirc^2 p & \rightarrow & \bigcirc^3 p \\ & \vdots & \end{array}$$

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- TEL models have the form

$$\begin{array}{ccccccccccc} \emptyset & & p & & \emptyset & & p & & \emptyset & & \\ \bullet & \longrightarrow & \bullet & \longrightarrow & \bullet & \longrightarrow & \bullet & \longrightarrow & \bullet & \longrightarrow & \ldots \end{array}$$

corresponding to LTL models of $\neg p \wedge \Box(\neg p \leftrightarrow \bigcirc p)$.

Some examples

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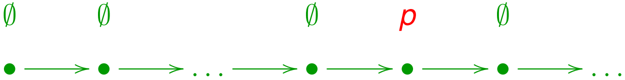

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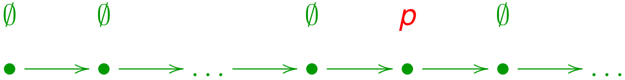

corresponding to LTL models of $\neg p \, \mathcal{U} \, (p \wedge \bigcirc \Box \neg p)$

An example

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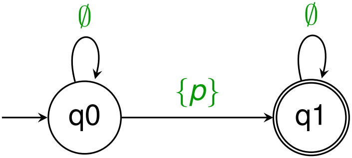

- Answer: using Büchi automata. An infinite-length word is accepted iff it visits some acceptance state infinitely often

Some examples

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- Now build some $\mathbf{H} < \mathbf{T}$ by removing one p at some point. But then $\langle \mathbf{H}, \mathbf{T} \rangle$ is also a model since $\mathbf{H}$ contains $\infty - 1 = \infty$ p's yet!

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- Therefore, $\Box\Diamond p$ alone has no TEL models.

Some examples

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$$\begin{aligned} & \square((\neg \bigcirc p \rightarrow p) \wedge (\bigcirc p \rightarrow p)) \\ \equiv\ & \square(\underbrace{\neg \bigcirc p \vee \bigcirc p}_{\top} \rightarrow p) \\ \equiv\ & \square p \end{aligned}$$

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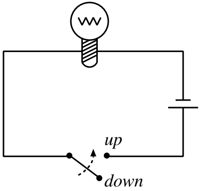
- So LTL models make p true forever, but we won't get TEL models!
- We can build a strictly smaller model with **H** where from some point on **T**, p becomes false forever

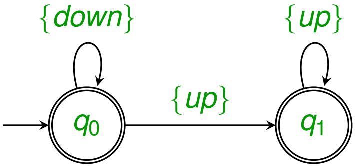
T	p	p		p	p	p	
	$\vert\vert$	$\vert\vert$		$\vert\vert$	$\cup$	$\cup$	
H	p	p		p	$\emptyset$	$\emptyset$	
	$\bullet \longrightarrow$	$\bullet \longrightarrow$	$\ldots \longrightarrow$	$\bullet \longrightarrow$	$\bullet \longrightarrow$	$\bullet \longrightarrow$	$\ldots$

Some examples

- Example 5: lamp switch again

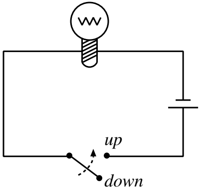


$up \vee down$	Initially
$\Box(up \wedge \neg \bigcirc down \rightarrow \bigcirc up)$	Inertia
$\Box(down \wedge \neg \bigcirc up \rightarrow \bigcirc down)$	Inertia
$\Box(up \vee \neg up)$	Choice



Some examples

- Example 5: lamp switch again



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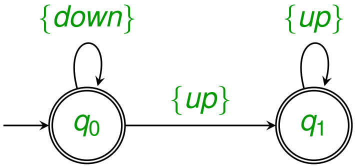

We never get $up \wedge down$

Once up is true, it remains so forever

Some examples

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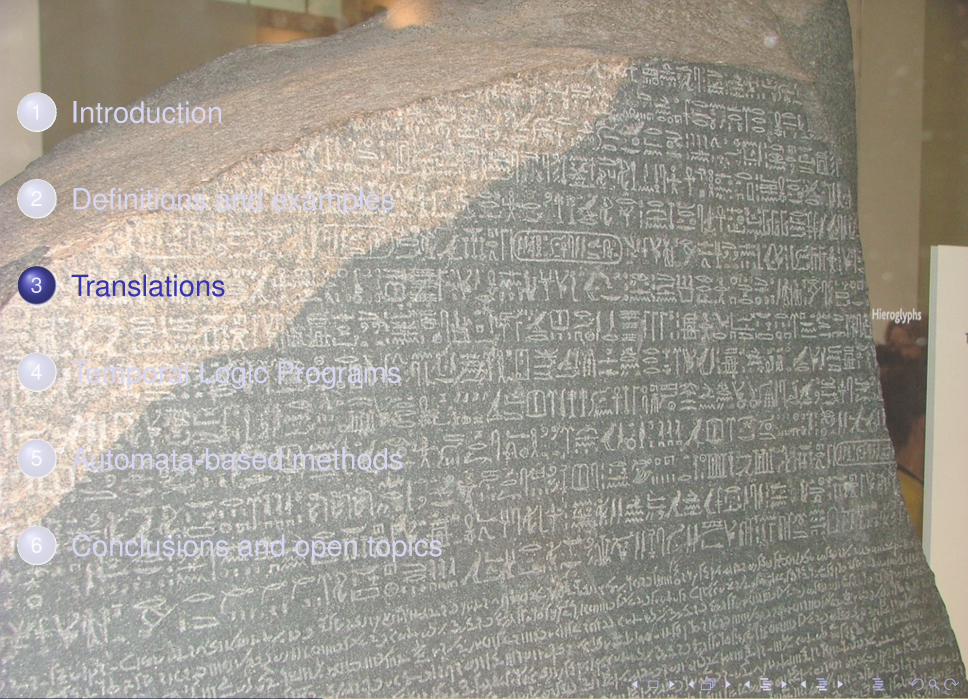
- Answer: natural translations to first-order and infinitary ...


Hieroglyphs

1. Enconding THT into LTL

- THT can be encoded into LTL, adding auxiliary atoms using the same translation of $\rightarrow$ from HT to classical logic
- Intuition: p will represent $p \in T$ whereas p' will mean $p \in H$

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Example

THT		**LTL**
$\Box(down \wedge \neg \bigcirc up \rightarrow \bigcirc down)$	$\equiv$	$\Box(up' \rightarrow up) \wedge \Box(down' \rightarrow down)$
		$\wedge\Box(down \wedge \neg \bigcirc up \rightarrow \bigcirc down)$
		$\wedge\Box(down' \wedge \neg \bigcirc up \rightarrow \bigcirc down')$

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  We know it holds for splittable temporal programs (see later)
- THT-satisfiability = PSPACE-complete [C_ & Demri 11]
  TEL-satisfiability = EXPSPACE-complete [Bozzelli & Pearce 15]

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- Note that $p \vee \neg p$ is alternate notation for a choice rule. We can selectively make a proposition behave as LTL/classical.

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- [C_, Diéguez, Vidal KR14]: Kamp's translation also sound for
  - from THT to Monadic Quantified HT with $<$
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- Kamp also proved the other direction MFO(<) $\mapsto$ LTL.
  Open question: Does it hold in our case?

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Not LTL-representable. Which kind of infinite sets of formulas?

Temporal Logic Programs

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- A temporal logic program (TLP for short) consists of

Definition (Temporal rule)

A temporal rule is either:

1. $Lit_1 \wedge \cdots \wedge Lit_n \rightarrow Lit_{n+1} \vee \cdots \vee Lit_m$
2. $\square(Lit_1 \wedge \cdots \wedge Lit_n \rightarrow Lit_{n+1} \vee \cdots \vee Lit_m)$
3. or an implication like $\square(\square p \rightarrow q)$ or like $\square(p \rightarrow \Diamond q)$
4. arbitrary constraints $\alpha \rightarrow \bot$

where p, q atoms and Lit_i expressions like $\bigcirc^i p$ or $\neg \bigcirc^i p$

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- Temporal stable models of a splittable TLP are LTL-representable: We can build loop formulas in LTL

Splittable TLPs

- [Aguado et al, LPNMR'11] introduces splittable TLPs splittable. Informally speaking: past does not depend on the future.

$$\Box(\underbrace{up}_{\text{past}} \wedge \underbrace{\neg \bigcirc down}_{\text{future}} \rightarrow \underbrace{\bigcirc up}_{\text{future}}) \qquad \text{Splittable}$$

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- System STeLP [C_ & Diéguez LPNMR11] uses loop formulas and backend model checker.

Automata-based methods

[C_ & Demri 2011]

Definition (Automata Based Computation Method)

LTL (i. e. total) models which do not have a strictly smaller $\langle H, T \rangle$

$\mathcal{A}_\varphi \quad \otimes \quad \overline{h(\mathcal{A}_{\varphi'})}$

- Intuition: $\mathcal{A}_{\varphi'}$ captures the $\langle \mathbf{H}, \mathbf{T} \rangle$ satisfying $\mathbf{H} < \mathbf{T}$

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- Büchi automata are closed w.r.t. complementation and intersection

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$\square\,(up \land \neg \bigcirc down \rightarrow \bigcirc up)\,.$

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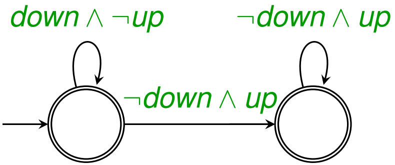

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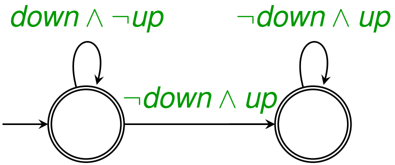

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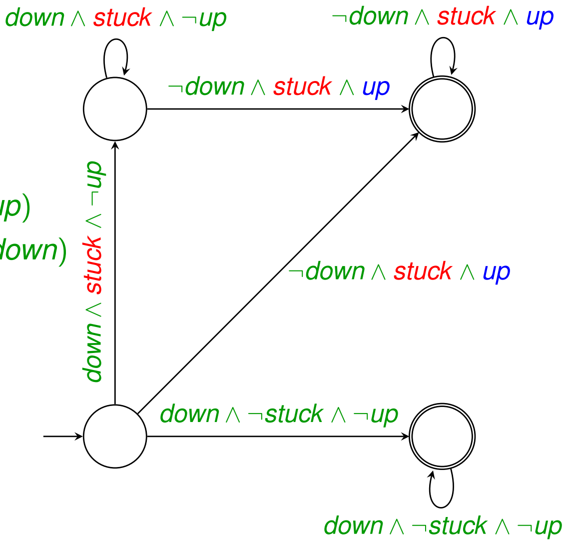

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When not THT-equivalent, ABSTEM provides a context that make both theories differ

Conclusions

- TEL = suitable framework for temporal reasoning + ASP
- Simple semantics thanks to just merging two logical formalisms: Equilibrium Logic + LTL.
- TEL does not “compete” with other ASP techniques: it complements them
  - when planning: non-existence of plans, temporal constraints
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  - checking strong equivalence
- It constitutes a new open field. Many open topics ...

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- Adding past operators:

  $$\Box(up \wedge \neg \bigcirc down \rightarrow \bigcirc up) \quad \text{versus} \quad \Box(\ominus up \wedge \neg down \rightarrow up)$$

  More natural when rule bodies refer to past

Open topics (wish list)

- Other temporal logics.
  Example: Equilibrium Logic+Dynamic LTL [Aguado et al. LPNMR13])
- New syntactic subclasses with satisfiability lower than EXPSPACE [Bozzelli & Pearce 15]
- Find a tableaux method for THT. Perhaps designing specific on-the-fly techniques
- Possible adaptation of Temporal Resolution [Fisher 91]
- Planning tool. Compare to planners using LTL control knowledge like TLPlan [Bacchus & Kabanza 00].
- Encoding action languages

Stable Models for Temporal Theories

Pedro Cabalar

Thanks for your attention!

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