Stable Models for Temporal Theories

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Joint work with KR group at

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and collaborators from other universities:

- David Pearce and Laura Bozzelli (Univ. Pol. Madrid, Spain),
- Stephane Demri (ENS de Cachan, France),
- Philippe Balbiani and Luis Fariñas (IRIT Toulouse, France)
1 Introduction

2 Definitions and examples

3 Translations

4 Temporal Logic Programs

5 Automata-based methods

6 Conclusions and open topics
Initial motivation

Formalizing **dynamic domains** was part of KR origins

- **Actions and Change**: temporal domains in first-order logic
  - Situation Calculus
  - Event Calculus
  - Features and Fluents
Initial motivation

Formalizing **dynamic domains** was part of KR origins

- **Actions and Change**: temporal domains in first-order logic
  - Situation Calculus
  - Event Calculus
  - Features and Fluents

- **Representational problems**: frame, Yale Shooting, …
  How to deal with **defaults** like **inertia**?
Initial motivation

The stress was put on Non-monotonic Reasoning (NMR)

- You said it’s a penguin?
- Well, it is not flying . . .
Initial motivation

[Image 104x192 to 154x235]
[Image 65x87 to 101x143]
[Image 24x46 to 143x83]

The stress was put on Non-monotonic Reasoning (NMR)

- You said it’s a penguin?
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[AIJ 1980] Circumscription, Default Logic, NM Modal logic
The stress was put on Non-monotonic Reasoning (NMR)

[AIJ 1980] Circumscription, Default Logic, NM Modal logic

Late 80’s, strong connection between LP - NMR
Logic Programming
Non-Monotonic Reasoning

- You said it’s a penguin?
- Well, it is not flying . . .
Example of correspondence:

\[
\begin{align*}
p & \leftarrow \text{not } q \\
r & \leftarrow p, \text{not } s
\end{align*}
\]

Stable models [Gelfond & Lifschitz 88] =

Default Logic [Reiter 80]
Example of correspondence:

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\[ r \leftarrow p, \text{not } s \]

\[ \text{Stable models} \]

\[ \text{Default Logic} \]

\[ \text{[Gelfond & Lifschitz 88]} \]

\[ \text{[Reiter 80]} \]

Why not using logic programs for action and change?
Example of correspondence:

\[
\begin{align*}
\text{LP} & : \quad p \leftarrow \neg q \\
& \quad r \leftarrow p, \neg s \\
\text{NMR} & : \quad \frac{\neg q}{p} \quad \frac{p : \neg s}{r}
\end{align*}
\]

Stable models

[Gelfond & Lifschitz 88]

Default Logic

[Reiter 80]

Why not using logic programs for action and change?

[Gelfond & Lifschitz, JLP 93]

*Representing Action and Change by Logic Programs*

Established a new methodology giving rise to . . .
Transition systems in Answer Set Programming (ASP)

Some nice features

✓ Elaboration tolerance: small changes in the problem $\Rightarrow$ small changes in representation

✓ Simple solution to frame, ramification and qualification problems
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✓ Incremental ASP exploits time index to reuse grounding/solving
Transition systems in ASP

But not thought for temporal reasoning

✗ Planning by iterative deepening with finite path length: we cannot prove non-existence of plan

✗ Reactive systems out of the scope: e.g. a network server must keep on running (potentially) forever

("Forgotten" reasoning task: verification of temporal properties. E.g. "At some point, fluent p will never change again")
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- Planning by *iterative deepening* with *finite path length*: we cannot prove *non-existence* of plan

- Reactive systems out of the scope:
  e.g. a *network server* must keep on running (potentially) forever

- ( Forgotten ) reasoning task: *verification of temporal properties.*
  E.g. “At some point, fluent $p$ will never change again”

- Existing formal methods for transition systems: *outside ASP*
Example

- Initially, a lamp switch can be *up* or *down*.

```prolog
time(0..n).
up(0), down(0).
```

\[\text{up}(T+1) : \neg \text{down}(T+1), \text{time}(T)\]

\[\text{down}(T+1) : \neg \text{up}(T+1), \text{time}(T)\]

\{\text{up}(T)\} : \text{time}(T)\]
Example

- Initially, a lamp switch can be *up* or *down*.
- By default, the switch state persists by inertia,

```
time(0..n).
up(0), down(0).
up(T+1) :- up(T), not down(T+1), time(T).
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```
Example

- Initially, a lamp switch can be \textit{up} or \textit{down}.
- By default, the switch state persists by inertia, but we can \textit{arbitrarily close} it at any moment.

\begin{verbatim}
transition_system

\text{time}(0..n).
\text{up}(0), \text{down}(0).

\text{up}(T+1) :- \text{up}(T), \neg \text{down}(T+1), \text{time}(T).
\text{down}(T+1) :- \text{down}(T), \neg \text{up}(T+1), \text{time}(T).
\{\text{up}(T)\} :- \text{time}(T).
\end{verbatim}
Examples of problems that cannot be solved using bounded time:

- Is there a reachable state with $up$ and $down$ false?
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- Is there a reachable state with \textit{up} and \textit{down} false?
- Once \textit{up} becomes true, does it remain so forever?
Examples of problems that cannot be solved using bounded time:

- Is there a reachable state with \textit{up} and \textit{down} false?
- Once \textit{up} becomes true, does it remain so forever?
- The switch cannot be closed infinitely often without eventually damaging the lamp
These topics typically covered by **(Modal) Temporal Logics**
Modal Temporal Logic

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- Mostly used in Theoretical Computer Science: algorithms, computability, complexity, formal verification
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- But, initially, not so much in Actions and Change
  [McCarthy97] “Modality, si! Modal logic, no!”
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Mostly used in Theoretical Computer Science: algorithms, computability, complexity, formal verification

But, initially, not so much in Actions and Change [McCarthy97] “Modality, si! Modal logic, no!”

Nowadays, temporal logics used in KR or planning, but difficult combination with NMR
Modal Temporal Logic

A simple and well-known case

Linear-time Temporal Logic (LTL)

□ (forever), ♦ (eventually), ○ (next), U (until)
Modal Temporal Logic

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✓ Decidable inference methods. Satisfiability: PSPACE-complete
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\( \Box \) (forever), \( \Diamond \) (eventually), \( \bigcirc \) (next), \( U \) (until)

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✓ Relation to other mathematical models:
  algebra, automata, formal languages
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✓ Fragment of First-Order Logic: [Kamp 68] LTL = Monadic FO (\(<\)
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✓ Model checking and verification of reactive systems
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✓ Relation to other mathematical models:
  algebra, automata, formal languages

✓ Fragment of First-Order Logic: [Kamp 68] LTL = Monadic FO (<)

✓ Model checking and verification of reactive systems

✓ Many uses in AI: planning, ontologies, multi-agent systems, ...
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× Monotonic: action domain representations manifest frame problem
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In model checking no worry on this: usually, logical description of **automaton states**
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Linear-time Temporal Logic (LTL)
□, ◊, ◯, U ...

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✗ NMR attempts for LTL: limited syntax, only for queries, control rules, etc. Not really embodied in LTL
Our proposal

Temporal Equilibrium Logic (TEL) [C_&Pérez 07]

TEL = ASP + LTL

- **ASP**: logical characterisation Equilibrium Logic [Pearce 96]
- **LTL**: We add temporal operators $\square$, $\diamond$, $\bigcirc$, $U$, $R$.

Result: **Temporal Stable Models** for any arbitrary LTL theory.
Initially, a lamp switch can be closed \((p)\) or open \((q)\).

By default, the switch state persists by inertia,
but we can arbitrarily close it at any moment.

\[
\begin{align*}
time(0..n). \\
up(0), \down(0). \\
up(T+1) & :\text{ up}(T), \text{ not } \down(T+1), \text{ time}(T). \\
\down(T+1) & :\text{ down}(T), \text{ not } \up(T+1), \text{ time}(T). \\
\{\up(T)\} & :\text{ time}(T).
\end{align*}
\]
Initially, a lamp switch can be closed \((p)\) or open \((q)\).

By default, the switch state persists by inertia, but we can **arbitrarily close** it at any moment.

\[
\begin{align*}
&\text{up} \lor \text{down} \\
\square(\square \text{up} &\leftarrow \text{up} \land \neg \square \text{down}) &\text{Initially} \\
\square(\square \text{down} &\leftarrow \text{down} \land \neg \square \text{up}) &\text{Inertia} \\
\square(\text{up} \lor \neg \text{up}) &\text{Choice}
\end{align*}
\]

**Idea:** LTL syntax, but keeping ASP semantics
Equilibrium Logic [Pearce96]: generalises stable models for arbitrary propositional theories.
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1. A non-classical monotonic (intermediate) logic called Here-and-There (HT)

\[ HT \text{ models} \]

\[ Classical \text{ models} \]
Equilibrium Logic [Pearce96]: generalises stable models for arbitrary propositional theories. Consists of:

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2. A selection of (certain) minimal models that yields nonmonotonicity
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Interpretation = pairs $\langle H, T \rangle$ of sets of atoms $H \subseteq T$

Example: $H = \{p, q\}$, $T = \{p, q, r, s\}$.

Intuition: There = perhaps true
Here = proved
p
r
q
w
t
Not there = false

When $H = T$ we have a classical model.
Here-and-There

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Here-and-There

- Interpretation = pairs \( \langle H, T \rangle \) of sets of atoms \( H \subseteq T \)

- Example: \( H = \{ p, q \}, T = \{ p, q, r, s \} \). Intuition:

- When \( H = T \) we have a classical model.
Here-and-There

Satisfaction of formulas

\[ \langle H, T \rangle \models \varphi \iff \text{“} \varphi \text{ is proved”} \]
Here-and-There

Satisfaction of formulas

\( \langle H, T \rangle \models \varphi \iff \text{“} \varphi \text{ is proved”} \)

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- \( \langle H, T \rangle \models p \text{ if } p \in H \) (for any atom \( p \))
Satisfaction of formulas

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- \[ \langle H, T \rangle \models p \text{ if } p \in H \quad \text{(for any atom } p \text{)} \]
- \[ \land, \lor \text{ as always} \]
Here-and-There

Satisfaction of formulas

\[ \langle H, T \rangle \models \varphi \iff \text{"} \varphi \text{ is proved} \]
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- \[ \langle H, T \rangle \models p \text{ if } p \in H \quad \text{(for any atom } p) \]
- \[ \land, \lor \text{ as always} \]
- \[ \langle H, T \rangle \models \varphi \rightarrow \psi \text{ if both} \]
  - \[ T \models \varphi \rightarrow \psi \text{ classically} \]
  - \[ \langle H, T \rangle \models \varphi \text{ implies } \langle H, T \rangle \models \psi \]
Satisfaction of formulas

\( \langle H, T \rangle \models \varphi \iff \text{“\( \varphi \) is proved”} \)

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- \( \langle H, T \rangle \models \varphi \rightarrow \psi \text{ if both} \)
  - \( T \models \varphi \rightarrow \psi \text{ classically} \)
  - \( \langle H, T \rangle \models \varphi \) implies \( \langle H, T \rangle \models \psi \)

- Negation \( \neg F \) is defined as \( F \rightarrow \bot \)

- \( \langle H, T \rangle \models \varphi \) implies \( T \models \varphi \) (proved implies potentially true)
Satisfaction of formulas

\[ \langle H, T \rangle \models \varphi \iff \text{“} \varphi \text{ is proved”} \]
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- Negation \( \neg F \) is defined as \( F \rightarrow \bot \)
- \[ \langle H, T \rangle \models \varphi \text{ implies } T \models \varphi \text{ (proved implies potentially true)} \]
- Relation to Gelfond & Lifschitz’s reduct:
  \[ \langle H, T \rangle \models P \text{ iff } H \models P^T \text{ classically} \]
A model $\langle T, T \rangle$ of $\Gamma$ is an equilibrium model iff

$$\text{there is no } H \subset T \text{ such that } \langle H, T \rangle \models \Gamma.$$ 

When this holds, $T$ is called a stable model.
Equilibrium models

Definition (Equilibrium/stable model)
A model $\langle T, T \rangle$ of $\Gamma$ is an equilibrium model iff

$$\text{there is no } H \subseteq T \text{ such that } \langle H, T \rangle \models \Gamma.$$ 

When this holds, $T$ is called a stable model.

In other words, all assumptions $T$ are eventually proved $H$.
Equilibrium logic

Well-known and understood, solid logical background, used in implementation, nice fundamental properties:
Equilibrium logic

Well-known and understood, solid logical background, used in implementation, nice fundamental properties:

- Two theories $\Gamma_1, \Gamma_2$ are strongly equivalent if $\Gamma_1 \cup \Gamma$ and $\Gamma_2 \cup \Gamma$ have the same equilibrium models for any $\Gamma$.

Strong equivalence of equilibrium theories = HT equivalence [Lifschitz, Pearce, Valverde 01].
Equilibrium logic

Well-known and understood, solid logical background, used in implementation, nice fundamental properties:

- Two theories $\Gamma_1, \Gamma_2$ are strongly equivalent if $\Gamma_1 \cup \Gamma$ and $\Gamma_2 \cup \Gamma$ have the same equilibrium models for any $\Gamma$.

  Strong equivalence of equilibrium theories = HT equivalence [Lifschitz, Pearce, Valverde 01].

- Captures all LP extensions with propositional connectives (also first-order [Pearce & Valverde 04]).

- Moreover, covers arbitrary formulas, in a very reasonable way:

  intuitionistic $\subset$ HT $\subset$ classical
Syntax = propositional plus

- □φ = “forever” φ
- ◊φ = “eventually” φ
- ◯φ = “next moment” φ
- φ U ψ = φ “until eventually” ψ
- φ R ψ = φ “release” ψ
Syntax = propositional plus

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- φ R ψ = φ “release” ψ

As we had with Equilibrium Logic:

1. A monotonic underlying logic: Temporal Here-and-There (THT)
2. An ordering among models. Select minimal models.
In standard LTL, interpretations are \( \infty \) sequences of sets of atoms.

\[
\begin{array}{cccccc}
\{p, q\} & \{p\} & \{q\} & \{\} & \{p, q\} & \ldots \\
0 & 1 & 2 & 3 & 4 & \\
\end{array}
\]
Sequences

- In standard LTL, interpretations are $\infty$ sequences of sets of atoms

$$\{p, q\} \rightarrow \{p\} \rightarrow \{q\} \rightarrow \{\} \rightarrow \{p, q\} \rightarrow \ldots$$

0 1 2 3 4

- In THT we will have $\infty$ sequences of HT interpretations

$$\ldots$$

0 1 2 3 4

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Stable Models for Temporal Theories

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Sequences

We define an ordering among sequences $H \leq T$ when

\[
T_0 \rightarrow T_1 \rightarrow T_2 \rightarrow \ldots \rightarrow T_i \rightarrow \ldots
\]

\[
U \upharpoonright \quad U \upharpoonright \quad U \upharpoonright \quad \quad \quad \quad \quad U \upharpoonright
\]

\[
H_0 \rightarrow H_1 \rightarrow H_2 \rightarrow \ldots \rightarrow H_i \rightarrow \ldots
\]
We define an ordering among sequences $H \leq T$ when

\[ T_0 \rightarrow T_1 \rightarrow T_2 \rightarrow \ldots \rightarrow T_i \rightarrow \ldots \]

\[ U \mid U \mid U \mid U \mid \]

\[ H_0 \rightarrow H_1 \rightarrow H_2 \rightarrow \ldots \rightarrow H_i \rightarrow \ldots \]

Definition (THT-interpretation)

is a pair of sequences of sets of atoms $\langle H, T \rangle$ with $H \leq T$. 
We define an ordering among sequences $H < T$ when

$$
T_0 \rightarrow T_1 \rightarrow T_2 \rightarrow \ldots \rightarrow T_i \rightarrow \ldots \\
U \mid U \mid U \mid U \\
H_0 \rightarrow H_1 \rightarrow H_2 \rightarrow \ldots \rightarrow H_i \rightarrow \ldots
$$

**Definition (THT-interpretation)**

is a pair of sequences of sets of atoms $\langle H, T \rangle$ with $H \leq T$.  

\[\]

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Temporal Here-and-There (THT)

\[ \langle H, T \rangle, i \models \varphi \iff \text{“}\varphi\text{ is proved at } i\text{”} \]
Temporal Here-and-There (THT)

\( \langle H, T \rangle, i \models \varphi \iff \text{"\varphi is proved at } i\text{"} \)

\( \langle T, T \rangle, i \models \varphi \iff \text{"\varphi potentially true at } i\text{"} \iff T, i \models \varphi \text{ in LTL} \)
Temporal Here-and-There (THT)

\[ \langle H, T \rangle, i \models \varphi \iff \text{“} \varphi \text{ is proved at } i \text{”} \]
\[ \langle T, T \rangle, i \models \varphi \iff \text{“} \varphi \text{ potentially true at } i \text{”} \iff T, i \models \varphi \text{ in LTL} \]

- An interpretation \( M = \langle H, T \rangle \) satisfies \( \alpha \) at situation \( i \), written \( M, i \models \alpha \)

<table>
<thead>
<tr>
<th>( \alpha )</th>
<th>( M, i \models \alpha ) when ...</th>
</tr>
</thead>
<tbody>
<tr>
<td>an atom ( p )</td>
<td>( p \in H_0 )</td>
</tr>
<tr>
<td>( \land, \lor )</td>
<td>as usual</td>
</tr>
<tr>
<td>( \varphi \rightarrow \psi )</td>
<td>( T, i \models \varphi \rightarrow \psi ) in LTL and ( \langle H, T \rangle, i \models \varphi ) implies ( \langle H, T \rangle, i \models \psi )</td>
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</table>
Temporal Here-and-There (THT)

$\langle H, T \rangle, i \models \varphi \iff \text{"}\varphi\text{ is proved at } i\text{"}$$

$\langle T, T \rangle, i \models \varphi \iff \text{"}\varphi\text{ potentially true at } i\text{"} \iff T, i \models \varphi\text{ in LTL}$

- An interpretation $M = \langle H, T \rangle$ satisfies $\alpha$ at situation $i$, written $M, i \models \alpha$

<table>
<thead>
<tr>
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<th>$M, i \models \alpha$ when . . .</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Box \varphi$</td>
<td>$(M, i+1) \models \varphi$</td>
</tr>
<tr>
<td>$\Diamond \varphi$</td>
<td>$\forall j \geq i, \ M, j \models \varphi$</td>
</tr>
<tr>
<td>$\Diamond U \psi$</td>
<td>$\exists j \geq i, \ M, j \models \psi$ and $\forall k$ s.t. $i \leq k &lt; j, \ M, k \models \varphi$</td>
</tr>
<tr>
<td>$\Diamond R \psi$</td>
<td>$\forall j \geq i, \ M, j \models \psi$ or $\exists k, i \leq k &lt; j, \ M, k \models \varphi$</td>
</tr>
</tbody>
</table>

- $M$ is a model of a theory $\Gamma$ when $M, 0 \models \alpha$ for all $\alpha \in \Gamma$
(Linear) Temporal Equilibrium Logic

\[ \bigcirc \varphi \]

\[ \varphi \]

[\[ \rightarrow \rightarrow \rightarrow \rightarrow \ldots \rightarrow \rightarrow \rightarrow \ldots \]
(Linear) Temporal Equilibrium Logic

- $\Diamond \varphi$

- $\square \varphi$

\[ \varphi \quad \varphi \quad \varphi \quad \varphi \quad \varphi \quad \varphi \]

\[ \bullet \rightarrow \bullet \rightarrow \bullet \rightarrow \ldots \rightarrow \bullet \rightarrow \ldots \]
(Linear) Temporal Equilibrium Logic

- $\Diamond \varphi$

- $\Box \varphi$

- $\lozenge \varphi$
**Linear** Temporal Equilibrium Logic

\[ \varphi \mathcal{U} \psi = \text{repeat } \varphi \text{ until (mandatorily) } \psi \]

\[ \bullet \rightarrow \bullet \rightarrow \bullet \rightarrow \ldots \rightarrow \bullet \rightarrow \bullet \rightarrow \ldots \]
(Linear) Temporal Equilibrium Logic

\[ \varphi U \psi = \text{repeat } \varphi \text{ until (mandatorily) } \psi \]

\[ \varphi \]

\[ \bullet \rightarrow \bullet \rightarrow \bullet \rightarrow \ldots \rightarrow \bullet \rightarrow \bullet \rightarrow \ldots \]
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\[ \phi \quad \phi \]

\[ \bullet \rightarrow \bullet \rightarrow \bullet \rightarrow \ldots \rightarrow \bullet \rightarrow \bullet \rightarrow \ldots \]
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\[ \bullet \rightarrow \bullet \rightarrow \bullet \rightarrow \ldots \rightarrow \bullet \rightarrow \bullet \rightarrow \ldots \]
(Linear) Temporal Equilibrium Logic

\[ \varphi \cup \psi = \text{repeat } \varphi \text{ until (mandatorily) } \psi \]

\[ \varphi \quad \varphi \quad \varphi \quad \varphi \]

\[ \bullet \rightarrow \bullet \rightarrow \bullet \rightarrow \ldots \rightarrow \bullet \rightarrow \bullet \rightarrow \ldots \]
\( \varphi \cup \psi = \text{repeat } \varphi \text{ until (mandatorily) } \psi \)

\[ \varphi \xrightarrow{} \varphi \xrightarrow{} \varphi \xrightarrow{} \ldots \xrightarrow{} \psi \]

\[ \bullet \xrightarrow{} \bullet \xrightarrow{} \bullet \xrightarrow{} \ldots \xrightarrow{} \bullet \xrightarrow{} \bullet \xrightarrow{} \ldots \]
(Linear) Temporal Equilibrium Logic

- \( \varphi \cup \psi = \text{repeat } \varphi \text{ until (mandatorily) } \psi \)

- \( \varphi \mathcal{R} \psi = \text{there is a } \varphi \text{ before any state without } \psi \)

\[ (M, i) \not\models \psi \]
(Linear) Temporal Equilibrium Logic

- $\varphi \mathcal{U} \psi = \text{repeat } \varphi \text{ until (mandatorily) } \psi$

  $\varphi \varphi \varphi \varphi \psi$

  $\bullet \rightarrow \bullet \rightarrow \bullet \rightarrow \ldots \rightarrow \bullet \rightarrow \bullet \rightarrow \ldots$

- $\varphi \mathcal{R} \psi = \text{there is a } \varphi \text{ before any state without } \psi$

  $\varphi \ \\ \ \ \ \ \ \ \ \ (M, i) \nvdash \psi$

  $\bullet \rightarrow \bullet \rightarrow \bullet \rightarrow \ldots \rightarrow \bullet \rightarrow \bullet \rightarrow \ldots$
Some valid THT formulas:

\[ \Diamond \varphi \leftrightarrow \top U \varphi \]

\[ \Box \varphi \leftrightarrow \bot R \varphi \]

\[ \Box (\varphi \otimes \psi) \leftrightarrow \Box \varphi \otimes \Box \psi \]

\[ \varphi U \psi \leftrightarrow \psi \lor (\varphi \land \Box (\varphi U \psi)) \]

\[ \varphi R \psi \leftrightarrow \psi \land (\varphi \lor \Box (\varphi R \psi)) \]

\[ \neg (\varphi U \psi) \leftrightarrow \neg \Box \neg \psi \]

\[ \Box \neg \varphi \leftrightarrow \neg \Box \varphi \]

\[ \neg (\varphi R \psi) \leftrightarrow \neg \varphi U \neg \psi \]

For \( \otimes = \land, \lor, \rightarrow, U, R \).
Temporal Here-and-There (THT)

Some valid THT formulas:

\[
\begin{align*}
\Diamond \varphi & \iff \top U \varphi \\
\square \varphi & \iff \bot R \varphi \\
\Box (\varphi \otimes \psi) & \iff \Box \varphi \otimes \Box \psi \\
\varphi U \psi & \iff \psi \lor (\varphi \land \Box (\varphi U \psi)) \\
\varphi R \psi & \iff \psi \land (\varphi \lor \Box (\varphi R \psi)) \\
\neg (\varphi U \psi) & \iff \neg \varphi R \neg \psi \\
\Box \neg \varphi & \iff \neg \Box \varphi \\
\neg (\varphi R \psi) & \iff \neg \varphi U \neg \psi
\end{align*}
\]

For \(\otimes = \land, \lor, \rightarrow, U, R\).

Axiomatization of THT: ongoing work [Balbiani & Diéguez 15]
Definition (Temporal Equilibrium Model)

of a theory $\Gamma$ is a model $M = \langle T, T \rangle$ of $\Gamma$ such that there is no $H < T$ satisfying $\langle H, T \rangle, 0 \models \Gamma$. 

Temporal Equilibrium Logic (TEL) is the logic induced by temporal equilibrium models.
**Temporal Equilibrium Models**

**Definition (Temporal Equilibrium Model)**

of a theory \( \Gamma \) is a model \( M = \langle T, T \rangle \) of \( \Gamma \) such that there is no \( H < T \) satisfying \( \langle H, T \rangle, 0 \models \Gamma \).

**Temporal Equilibrium Logic (TEL)** is the logic induced by temporal equilibrium models.

**Definition (Temporal Stable Model)**

\( T \) is a temporal stable model of a theory \( \Gamma \) iff \( \langle T, T \rangle \) is a temporal equilibrium model of \( \Gamma \).
Some examples

Example 1: TEL models of $\Box(\neg p \rightarrow \bigcirc p)$. It’s like an infinite program:

\[
\begin{align*}
\neg p & \rightarrow \bigcirc p \\
\neg \bigcirc p & \rightarrow \bigcirc^2 p \\
\neg \bigcirc^2 p & \rightarrow \bigcirc^3 p \\
\vdots
\end{align*}
\]
Some examples

- Example 1: TEL models of $\Box(\neg p \rightarrow \Diamond p)$. It’s like an infinite program:

\[
\neg p \rightarrow \Diamond p \\
\neg \Diamond p \rightarrow \Diamond^2 p \\
\neg \Diamond^2 p \rightarrow \Diamond^3 p \\
\vdots
\]

- TEL models have the form

\[
\emptyset \quad p \quad \emptyset \quad p \quad \emptyset \\
\bullet \rightarrow \bullet \rightarrow \bullet \rightarrow \bullet \rightarrow \bullet \rightarrow \bullet \rightarrow \ldots
\]

corresponding to LTL models of $\neg p \land \Box(\neg p \leftrightarrow \Diamond p)$.
Example 2: consider TEL models of $\lozenge p$
Some examples

Example 2: consider TEL models of $\Diamond p$

is like $p \lor \Box p \lor \Box \Box p \lor \ldots$
Some examples

Example 2: consider TEL models of $\diamond p$

is like $p \lor \Box p \lor \Box \Box p \lor \ldots$

TEL models have the form

$$
\emptyset \quad \emptyset \quad \emptyset \quad \emptyset \quad p \quad \emptyset
$$

\[ \quad \rightarrow \quad \rightarrow \quad \ldots \quad \rightarrow \quad \rightarrow \quad \rightarrow \quad \rightarrow \quad \ldots \]
Some examples

- Example 2: consider TEL models of $\Diamond p$
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  TEL models have the form

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  $\bullet \rightarrow \bullet \rightarrow \ldots \rightarrow \bullet \rightarrow \bullet \rightarrow \bullet \rightarrow \ldots$

  corresponding to LTL models of $\neg p \mathcal{U} (p \land \Box \Box \neg p)$
In ASP terms, how can we represent temporal stable models? 
infinite long! infinitely many!

Answer: using Büchi automata. An infinite-length word is accepted iff it visits some acceptance state infinitely often.
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Some examples

- Example 3: consider TEL models of $\Box \Diamond p$
- In LTL this means $p$ occurs infinitely often.
Some examples

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- So take any LTL model $T$ like that, i.e., $\langle T, T \rangle$ is a total THT model.
Some examples

- Example 3: consider TEL models of $\Box \Diamond p$
- In LTL this means $p$ occurs infinitely often.
- So take any LTL model $T$ like that, i.e., $\langle T, T \rangle$ is a total THT model.
- Now build some $H < T$ by removing one $p$ at some point. But then $\langle H, T \rangle$ is also a model since $H$ contains $\infty - 1 = \infty$ $p$’s yet!
Some examples

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- Now build some $H < T$ by removing one $p$ at some point. But then $\langle H, T \rangle$ is also a model since $H$ contains $\infty - 1 = \infty$ $p$'s yet!
- Therefore, $\Box \Diamond p$ alone has no TEL models.
Some examples

- Example 4: consider TEL models of the pair of formulas

\[
\square (\neg \lozenge p \to p) \\
\square (\lozenge p \to p)
\]

Curiosity: implications go backwards in time

This is LTL-equivalent to:

\[
\square (\neg \lozenge p \to \neg \lozenge p) \land (\lozenge p \to \lozenge p) \equiv \square (\neg \lozenge p \lor \lozenge p \land \top) \to p 
\]

\[
\equiv \square p
\]

Pedro Cabalar (Department of Computer Science, University of Corunna (Spain)
cabalar@udc.es

Stable Models for Temporal Theories

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Some examples

- Example 4: consider TEL models of the pair of formulas

\[
\square (\neg \bigcirc p \rightarrow p) \\
\square (\bigcirc p \rightarrow p)
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- Curiosity: implications go **backwards in time**
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\[ \square(\neg \bigcirc p \rightarrow p) \]
\[ \square(\bigcirc p \rightarrow p) \]

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Some examples

- Example 4: consider TEL models of the pair of formulas

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- Curiosity: implications go backwards in time

This is LTL-equivalent to:

\[ \square((\neg \bigcirc p \rightarrow p) \land (\bigcirc p \rightarrow p)) \]
\[ \equiv \square(\neg \bigcirc p \lor \bigcirc p \rightarrow p) \]
\[ \equiv \square p \]
Some examples

- Example 4: consider TEL models of the pair of formulas

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\Box (\neg \Diamond p \rightarrow p) \\
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- So LTL models make \( p \) true forever,
Some examples

- Example 4: consider TEL models of the pair of formulas

\[ \Box(\neg \Diamond p \rightarrow p) \]
\[ \Box(\Diamond p \rightarrow p) \]

- So LTL models make \( p \) true forever, but we won’t get TEL models!
Some examples

- Example 4: consider TEL models of the pair of formulas
  \[ \Box (\neg \Diamond p \rightarrow p) \]
  \[ \Box (\Diamond p \rightarrow p) \]

  So LTL models make \( p \) true forever, but we won’t get TEL models!

  We can build a strictly smaller model with \( H \) where from some point on \( T \), \( p \) becomes false forever

\[
\begin{array}{cccccc}
\text{T} & p & p & p & p & p \\
\| & \| & \| & \| & \mathcal{U} & \mathcal{U} \\
\text{H} & p & p & p & \emptyset & \emptyset \\
\end{array}
\]

\[ \bullet \rightarrow \bullet \rightarrow \ldots \rightarrow \bullet \rightarrow \bullet \rightarrow \bullet \rightarrow \ldots \]
Some examples

Example 5: lamp switch again

\[ \text{Initially} \quad \square (up \land \neg \Diamond down) \rightarrow \Diamond up \]
\[ \text{Inertia} \quad \square (down \land \neg \Diamond up) \rightarrow \Diamond down \]
\[ \text{Inertia} \quad \square (up \lor \neg up) \]

We never get \( up \land down \)

Once \( up \) is true, it remains so forever

\[ \{down\} \rightarrow \{up\} \]
\[ \text{Choice} \quad q_0 \rightarrow q_1 \]
Some examples

Example 5: lamp switch again

\[ \begin{align*}
\square (up \land \neg \lozenge down) & \rightarrow \lozenge up) \\
\square (down \land \neg \lozenge up) & \rightarrow \lozenge down \\
\square (up \lor \neg up) & \text{Choice}
\end{align*} \]

Initially

Inertia

Inertia

We never get \( up \land down \)

Once \( up \) is true, it remains so forever
Some examples

- Reasonable behavior when theories “look like” logic programs

\[ \text{e.g.} \quad p \land (\neg \square \diamond p \rightarrow \diamond (p \cup q)) \]
Some examples

- Reasonable behavior when theories “look like” logic programs

- But what happens with arbitrary temporal formulas?
  e.g. $\Diamond p \land (\neg \Box \Diamond q \rightarrow \Diamond (p \mathcal{U} q))$
Some examples

- Reasonable behavior when theories “look like” logic programs

- But what happens with arbitrary temporal formulas?
  e.g. $\lozenge p \land (\neg \Box \lozenge q \rightarrow \lozenge (p \mathcal{U} q))$

- Answer: natural translations to first-order and infinitary ...
1. Introduction
2. Definitions and examples
3. Translations
4. Temporal Logic Programs
5. Automata-based methods
6. Conclusions and open topics
1. Encoding THT into LTL

- THT can be encoded into LTL, adding auxiliary atoms using the same translation of \( \rightarrow \) from HT to classical logic.

- Intuition: \( p \) will represent \( p \in T \) whereas \( p' \) will mean \( p \in H \).
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Example

<table>
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<tr>
<th>THT</th>
<th>LTL</th>
</tr>
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<tbody>
<tr>
<td>$\square((\text{down} \land \neg \lozenge \text{up} \rightarrow \lozenge \text{down})$</td>
<td>$\square((\text{up}' \rightarrow \text{up}) \land \square((\text{down}' \rightarrow \text{down})$</td>
</tr>
<tr>
<td>$\land \square((\text{down} \land \neg \lozenge \text{up} \rightarrow \lozenge \text{down})$</td>
<td>$\land \square((\text{down}' \land \neg \lozenge \text{up} \rightarrow \lozenge \text{down}')$</td>
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1. Encoding THT into LTL

**Warning:** this does not mean that we can encode **Temporal Stable Models** as models of an LTL theory!

THT-satisfiability = PACE

TEL-satisfiability = E^{XP_S}_{PACE} [Bozzelli & Pearce 15]
1. Encoding THT into LTL

- **Warning**: this does not mean that we can encode **Temporal Stable Models** as models of an LTL theory!

- This is an open question
  (failed attempt [C_ & Diéguez, ASPOCP’14])

We know it holds for **splittable temporal programs** (see later)
1. Encoding THT into LTL

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- This is an open question (failed attempt [C_ & Diéguez, ASPOCP’14])
- We know it holds for splittable temporal programs (see later)

- THT-satisfiability = PSPACE-complete [C_ & Demri 11]
- TEL-satisfiability = EXPSPACE-complete [Bozzelli & Pearce 15]
Encoding LTL into THT is straightforward. Add the excluded middle axiom for all atom $p$:

$$\Box(p \lor \neg p)$$
Encoding LTL into THT is straightforward. Add the excluded middle axiom for all atom $p$:

$$\Box(p \lor \neg p)$$

Note that $p \lor \neg p$ is alternate notation for a choice rule. We can selectively make a proposition behave as LTL/classical.
3. TEL into First-Order Equilibrium Logic

- Most modal logics, natural translation into First-Order Logic (FOL)

<table>
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<tr>
<td>♦ up</td>
<td>→ ∃(x ≥ 0 ∧ up(x))</td>
</tr>
<tr>
<td>♦□ up</td>
<td>→ ∃(x ≥ 0 ∧ ∀y(y ≥ x → up(y)))</td>
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[Kamp 68]: Kamp’s translation also sound from THT to Monadic Quantified HT with from TEL to Monadic Quantified Equilibrium Logic (MFO(<)). Kamp also proved the other direction MFO(<) ↦→ LTL.

Open question: Does it hold in our case?
3. TEL into First-Order Equilibrium Logic

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- [Kamp 68]: from LTL into MFO(<), monadic FOL plus < relation

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- Rather than ∀ or ∃ we use infinitary conjunction and disjunction

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<tbody>
<tr>
<td>□p</td>
<td>∀x (x ≥ 0 → p(x))</td>
<td>p ∧ □p ∧ □²p ∧ ...</td>
</tr>
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<td>◊p</td>
<td>∃x (x ≥ 0 ∧ p(x))</td>
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✓ Propositional signature: each ‘□¬p’ understood as an atom.
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✓ Propositional signature: each ‘□ᵢp’ understood as an atom.

✗ But even adding excluded middle, infinitary logic more expressive than LTL or MFO(⟨)

\[ \{□ᵢp \mid i ≥ 0 \text{ and } mod(i, 2) = 0\}^\wedge \equiv p ∧ □²p ∧ □⁴p ∧ □⁶p ∧ ... \]

Not LTL-representable.
4. TEL into Infinitary Equilibrium Logic

- A similar correspondence can be proved for Infinitary Equilibrium Logic [Harrison et al, ASPOCP’14]

- Rather than $\forall$ or $\exists$ we use infinitary conjunction and disjunction

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<tr>
<td>$\Box p$</td>
<td>$\forall x \ (x \geq 0 \rightarrow p(x))$</td>
<td>$p \land \Diamond p \land \Diamond^2 p \land \ldots$</td>
</tr>
<tr>
<td>$\Diamond p$</td>
<td>$\exists x \ (x \geq 0 \land p(x))$</td>
<td>$p \lor \Diamond p \lor \Diamond^2 p \lor \ldots$</td>
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✓ Propositional signature: each $\Diamond^i p$ understood as an atom.

✗ But even adding excluded middle, infinitary logic more expressive than LTL or MFO($\prec$)

$\{\Diamond^i p \mid i \geq 0 \text{ and } \text{mod}(i, 2) = 0\}^\land$

$\equiv p \land \Diamond^2 p \land \Diamond^4 p \land \Diamond^6 p \land \ldots$

Not LTL-representable. Which kind of infinite sets of formulas?
Temporal Logic Programs

- THT theories can be reduced to a normal form: temporal logic programs TLPs [C_, JELIA’10].
- Structure preserving transformation introducing auxiliary atoms.
Temporal Logic Programs

- THT theories can be reduced to a normal form: temporal logic programs TLPs [C_, JELIA’10].
- Structure preserving transformation introducing auxiliary atoms.
- A temporal logic program (TLP for short) consists of

**Definition (Temporal rule)**

A temporal rule is either:

1. \( \text{Lit}_1 \land \cdots \land \text{Lit}_n \rightarrow \text{Lit}_{n+1} \lor \cdots \lor \text{Lit}_m \)
2. \( \Box(\text{Lit}_1 \land \cdots \land \text{Lit}_n \rightarrow \text{Lit}_{n+1} \lor \cdots \lor \text{Lit}_m) \)
3. or an implication like \( \Box(\Box p \rightarrow q) \) or like \( \Box(p \rightarrow \Diamond q) \)
4. arbitrary constraints \( \alpha \rightarrow \bot \)

where \( p, q \) atoms and \( \text{Lit}_i \) expressions like \( \bigcirc^i p \) or \( \neg \bigcirc^i p \)
Splittable TLPs


\[
\square(up \land \neg \Diamond down \rightarrow \Diamond up)
\]

Splittable

\[
\square(\Diamond p \land \rightarrow p)
\]

Non-splittable

Our switch example theory was splittable.
Splittable TLPs


\[ \square(\text{up} \land \neg \square \text{down} \rightarrow \square \text{up}) \]  

Splittable

\[ \square (\square \text{p} \land \rightarrow \text{p}) \]  

Non-splittable

- Our switch example theory was splittable.

- Temporal stable models of a splittable TLP are LTL-representable: We can build loop formulas in LTL.
Splittable TLPs


\[ \Box (\text{up} \land \neg \Box \text{down}) \rightarrow \Box \text{up} \] Splittable

\[ \Box (\Box p \land \rightarrow p) \] Non-splittable

- Our switch example theory was splittable.
- Temporal stable models of a splittable TLP are LTL-representable: We can build loop formulas in LTL
- System \textbf{STeLP} [C_ & Diéguez LPNMR11] uses loop formulas and backend model checker.
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Automata-based methods

[C_ & Demri 2011]

Definition (Automata Based Computation Method)

LTL (i.e., total) models which do not have a strictly smaller $\langle H, T \rangle$

$A_\varphi \otimes h(A_{\varphi'})$

Intuition: $A_{\varphi'}$ captures the $\langle H, T \rangle$ satisfying $H < T$
Automata-based methods

[C_ & Demri 2011]

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\[ \mathcal{A}_\varphi \otimes h(\mathcal{A}_\varphi') \]

- **Intuition:** \( \mathcal{A}_\varphi' \) captures the \( \langle H, T \rangle \) satisfying \( H < T \)

- We use the \( \varphi^* \) translation and force non-LTL models.

**Example:** if \( \varphi = \diamond up \) then

\[ \varphi' = \diamond up' \land \square (up' \rightarrow up) \land \diamond (up \land \neg up') \]
Automata-based methods

[C_ & Demri 2011]

Definition (Automata Based Computation Method)

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Büchi automata are closed w.r.t. complementation and intersection
Example of non-splittable theory

\[ up \lor down. \]
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\[ \square (up \lor \neg up) \]

\[ down \land \neg up \]
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ABSTEM: obtains temporal stable models for arbitrary theories

- It also allows checking different types of equivalence
  - LTL equivalence
  - Weak equivalence (same temporal stable models)
  - Strong equivalence

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\[ \phi_1 \text{ and } \phi_2 \text{ are strongly equivalent iff they are THT equivalent.} \]

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2 Definitions and examples

3 Translations

4 Temporal Logic Programs

5 Automata-based methods

6 Conclusions and open topics
Conclusions

- TEL = **suitable framework** for temporal reasoning + ASP

- Simple semantics thanks to just **merging two logical formalisms**: Equilibrium Logic + LTL.

- TEL does not “compete” with other ASP techniques: it **complements** them
  - when planning: non-existence of plans, temporal constraints
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Conclusions

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- Simple semantics thanks to just merging two logical formalisms: Equilibrium Logic + LTL.
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  - when planning: non-existence of plans, temporal constraints
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  - checking strong equivalence
- It constitutes a new open field. Many open topics . . .
Open topics (wish list)

- Complete Axiomatisation of Temporal Here-and-There (almost done [Balbiani & Diéguez])

Can we represent the temporal stable models of $\Gamma$ as LTL models? Our conjecture: positive

THT vs Quantified HT (QHT): analogous to Kamp's theorem for THT and monadic QHT with $<$

Our conjecture: negative. It seems we cannot move $q$ out of $\exists x$ in $\exists x ( (p(x) \rightarrow q) \land r(x) )$

Adding past operators: $\Box (\uparrow \land \neg \downarrow \rightarrow \uparrow)$ versus $\Box (\neg \uparrow \land \neg \downarrow \rightarrow \uparrow)$

More natural when rule bodies refer to past

Pedro Cabalar (Department of Computer Science University of Corunna (Spain) cabalar@udc.es)
Open topics (wish list)

- Complete **Axiomatisation of Temporal Here-and-There** (almost done [Balbiani & Diéguez])

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- **Adding past operators:**

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  \Box (up \land \neg \Box down \to \Box up) \quad \text{versus} \quad \Box (\exists up \land \neg down \to up)\]

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Open topics (wish list)

- Other temporal logics. Example: Equilibrium Logic+Dynamic LTL [Aguado et al. LPNMR13]

- New **syntactic subclasses** with satisfiability lower than \(\text{EXPSPACE}\) [Bozzelli & Pearce 15]

- Find a **tableaux method** for THT. Perhaps designing specific on-the-fly techniques

- Possible adaptation of Temporal Resolution [Fisher 91]

- **Planning tool**. Compare to planners using LTL control knowledge like TLPlan [Bacchus & Kabanza 00].

- Encoding action languages
Stable Models for Temporal Theories

Pedro Cabalar

Thanks for your attention!

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