Fuzzy sets and many-valued logics in foundations of quantum mechanics

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Since 1936 Birkhoff and von Neumann paper "The logic of quantum mechanics" it is generally accepted that a "quantum logic", i.e., a set of dichotomic ("yes-no") statements about properties of a quantum object has an algebraic structure of an orthomodular lattice. Therefore, a "quantum logic" is usually thought to be a two-valued logic, non-classical because of lack of distributivity. In 1972 Maciej Mączyński proved that any orthomodular poset, therefore also an orthomodular lattice, possessing an order determining set of states (only such structures are interesting from the physical point of view), can be isomorphically represented as a specific family of functions that take values in the interval [0,1].

A fuzzy set is an object characterized by a membership function that takes values in the interval [0,1]. It followed that in 1994 Mączyński result was extended by the author to a fuzzy set representation of Birkhoff-von Neumann "quantum logics". Since fuzzy sets remain in the same relation to infinite-valued logics as traditional sets to 2-valued logic, it was straightforward to further extend this representation to infinite-valued logic.

As a result, any "quantum logic" in the Birkhoff-von Neumann sense possessing an order determining set of states, e.g., the lattice of (projections onto) closed subspaces of a Hilbert space used to describe a quantum object, can be isomorphically represented as a specific infinite-valued Łukasiewicz logic endowed with partially defined conjunction and disjunction.

Such representation forces treating properties of quantum objects, before they are measured, as possessed in the "fuzzy" or "many-valued" sense. This eliminates numerous paradoxes which, as a GHZ paradox, follow from the assumed definite possessment or non-possessment by a quantum object all of its properties prior to their measurement, and also lays the foundation stone for a new "many-valued" interpretation of quantum mechanics.