"Locally Projective Graphs, Majorana Theory and the Monster Group "

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Let Γ be a graph with an edge-transitive automorphism group G such that (a) Γ is bipartite with vertices in one part having valency $\alpha \in \{2,3\}$ (they are called *lines*), and the vertices in the other part having valency $2^n - 1$ for some $n \geq 2$ (they are called *points*); (b) the stabilizer in G of a point induces the natural doubly transitive action of $L_n(2)$ on the set of lines containing this point. The ultimate goal is to classify the amalgams $\{G(p), G(l)\}$ formed by the stabilizers of adjacent point-line pairs (p,l). For $n = 2$ there 15 such amalgam as proved in the ground breaking paper by D.Goldschmidt. For every Goldschmitd's amalgam the order of $G(p)$ divides $2^7 \cdot 3$ and the largest amalgam is realized in the automorphism group of the Mathieu group M_{12}. For $n \geq 3$ the known examples come from point-line incident graphs of classical and sporadic flag-transitive geometries. The classical examples are associated with the linear and symplectic geometries over $GF(2)$ along with the Hamming graphs over the alphabet with 3 letters and automorphism group $S_3 \wr L_n(2)$. The sporadic examples include the exceptional A_7-geometry, Cooperstein's geometry of $G_2(3)$, and tilde geometries of the Mathieu group M_{24}, the Held group He, the Conway group Co_1, culminating at the Monster group M. The similar problem with lines having valency 2 was accomplished by S.V.Shpectorov and the author with essential use of V.I. Trofimov's results announced in 1991 and published in numerous subsequent papperse along with the classification of Petersen geometries. A special case of $n = 3$ problem with lines of valency 3 was settled was accomplished recently, where the amalgams coming from M_{24}- and He-geometries were characterized (and proved to be isomorphic to each other), and a new amalgam realized in A_{16} was discovered. Our general strategy is to recover a geometry from the graph and to apply the amalgam method to tackle down the possible residues. The classification of tilde geometries by the authors of is expected to be the final accord.