VARIABILI ALEATORIE CONTINUE

DISCRETE

(1) $X : \Omega \to \mathbb{R}$, Ω SPAZIO CAMP DISCRETO

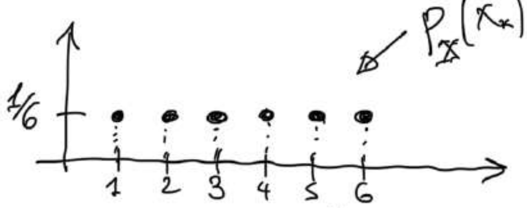
(2) DENSITÀ DISCRETA

$$p_X(x_k) = P(X = x_k) \geq 0$$

$$\{x_k\} \subseteq \mathbb{R}$$

È UNA PROBABILITÀ

(3) $$\sum_k p_X(x_k) = 1$$



(4) $$P(2 \leq X \leq 5) = \sum_{k=2}^{5} p_X(k)$$

(5) LEGGE DISCRETA

$$P(X \in I) = \sum_{x_k \in I} p_X(x_k)$$

(6) VALORE ATTESO

$$E(X) = \sum_k x_k \cdot p_X(x_k)$$

(7) VARIANZA

$$Var(X) = E(X^2) - E(X)^2$$

$$= \sum_k x_k^2 \cdot p_X(x_k) - E(X)^2$$

CONTINUE

(1) $X : \Omega \to \mathbb{R}$, Ω SPAZIO CAMP CONTINUO

(2) DENSITÀ CONTINUA

$$f_X(t) \geq 0$$

$$t \in \mathbb{R}$$

<u>NON È LA PROB.</u> PER OGNI SINGOLO VALORE

(3) $$\int_{-\infty}^{+\infty} f_X(t)\,dt = 1$$

(4) $$P(2 \leq X \leq 5) = \int_2^5 f_X(t)\,dt$$

(5) LEGGE CONTINUA

$$P(X \in I) = \int_I f_X(t)\,dt$$

(6) VALORE ATTESO

$$E(X) = \int_{-\infty}^{+\infty} t \cdot f_X(t)\,dt$$

(7) VARIANZA

$$Var(X) = E(X^2) - E(X)^2$$

$$= \int_{-\infty}^{+\infty} t^2 \cdot f_X(t)\,dt - E(X)^2$$

SE VOGLIAMO CALCOLARE LA $P(X \leq 2)$

$$\underbrace{\sum_k P(X \leq 2)}_{\text{AD ESEMPIO PER UNA BINOMIALE (DISCRETO)}}$$

$$\boxed{F(2)} := \int_{-\infty}^{2} f_X(t)\,dt$$

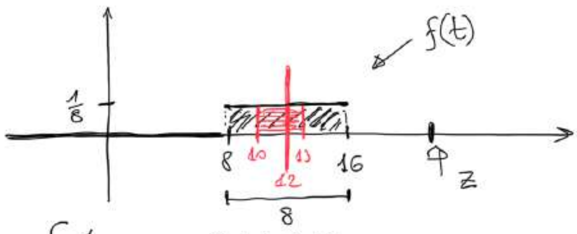
IN GENERALE

$$F(t) := P(X \leq t)$$

SI CHIAMA <u>FUNZIONE DI RIPARTIZIONE</u> DELLA V. A.

ESEMPIO (DENSITÀ UNIFORME)

(a) CHI È LA FUNZIONE DI DENSITÀ



$$f(t) = \begin{cases} \frac{1}{8} & \text{SE } 8 \leq t \leq 16 \\ 0 & \text{ALTRIMENTI} \end{cases}$$

$$P(10 \leq X \leq 13) := \int_{10}^{13} f(t)\,dt = \int_{10}^{13} \frac{1}{8}\,dt = \frac{1}{8}t\Big]_{10}^{13} =$$

$$= \frac{13}{8} - \frac{10}{8} = \boxed{\frac{3}{8}}$$

(b) $$P(11 \leq X \leq 15) = \int_{11}^{15} \frac{1}{8}\,dt = \frac{1}{8}t\Big]_{11}^{15} = \frac{15}{8} - \frac{11}{8} = \frac{4}{8} = \frac{1}{2}$$

$$P(X \geq 14) = \int_{14}^{+\infty} f(t)\,dt = \int_{14}^{16} \frac{1}{8}\,dt + \underbrace{\int_{16}^{+\infty} 0\,dt}_{=0} = \frac{1}{8}t\Big]_{14}^{16} =$$

$$= \frac{16}{8} - \frac{14}{8} = \frac{2}{8} = \frac{1}{4}$$

(c) $$E(X) = \int_{-\infty}^{+\infty} t \cdot f(t)\,dt = \int_{8}^{16} t \cdot \frac{1}{8}\,dt = \frac{1}{8}\int_{8}^{16} t\,dt = \frac{1}{8}\left[\frac{t^2}{2}\right]_8^{16} =$$

$$= \frac{1}{8}\left[128 - 32\right] = \frac{96}{8} = 12$$

$$Var(X) = E(X^2) - E(X)^2 = \int_{-\infty}^{+\infty} t^2 \cdot f(t)\,dt - 144 =$$

$$= \int_{8}^{16} t^2 \cdot \frac{1}{8}\,dt - 144 = \frac{1}{8}\left[\frac{t^3}{3}\right]_8^{16} - 144 =$$

$$= \frac{1}{8}\left[\frac{16^3}{3} - \frac{8^3}{3}\right] - 144 = \frac{1}{8}\left[\frac{2^{12} - 2^9}{3}\right] - 144$$

$$= \frac{1}{\cancel{2^3}} \frac{2^{\cancel{9}\,6}(2^3 - 1)}{3} - 144 = \frac{64 \cdot 7}{3} - 144 = \boxed{5.33}$$

(d) $$F(z) = P(X \leq z) = \int_{-\infty}^{z} f(t)\,dt$$

f DIVIDE $\mathbb{R}$ IN 3 PEZZI : (1) $(-\infty, 8)$; (2) $[8, 16]$; (3) $(16, +\infty)$

(1) SE $z < 8$ $\quad F(z) = \int_{-\infty}^{z} 0\,dt = 0$

(2) SE $8 \leq z \leq 16$ $\quad F(z) = \int_{-\infty}^{z} f(t)\,dt = \underbrace{\int_{-\infty}^{8} 0\,dt}_{=0} + \int_{8}^{z} \frac{1}{8}\,dt =$

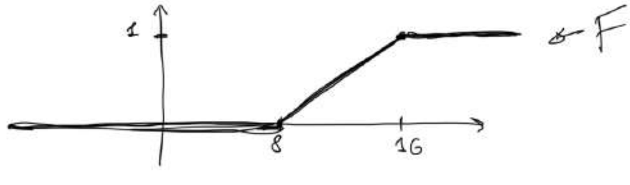
$$= \left[\frac{1}{8}t\right]_8^z = \boxed{\frac{z}{8} - 1}$$

(3) SE $z > 16$ $\quad F(z) = \int_{-\infty}^{z} f(t)\,dt = \underbrace{\int_{-\infty}^{8} 0\,dt}_{=0} + \int_{8}^{16} \frac{1}{8}\,dt + \underbrace{\int_{16}^{z} 0\,dt}_{0} =$

$$= \frac{1}{8}t\Big]_8^{16} = \frac{16}{8} - \frac{8}{8} = \frac{8}{8} = \boxed{1}$$

$$F(z) = \begin{cases} 0 & \text{SE } z < 8 \\ \frac{z}{8} - 1 & \text{SE } 8 \leq z \leq 16 \\ 1 & \text{SE } z > 16 \end{cases}$$

RAPPRESENTA L'ACCUMULAZIONE DELLA PROBABILITÀ



GAUSSIANA STANDARD $Z \sim N(0,1)$

normale $\rightarrow \mu \rightarrow \sigma^2$

$$\boxed{Z = \frac{X-\mu}{\sigma}}$$, X È UNA QUALSIASI GAUSSIANA CON MEDIA μ E VARIANZA σ^2

$$X = \underbrace{\sigma Z + \mu}$$

CHI È LA FUNZIONE DI RIPARTIZIONE DI X?

$$F_X(t) = P(X \leq t) = P(\underbrace{\sigma Z + \mu \leq t}) =$$

$$= \underbrace{P\left(Z \leq \frac{t-\mu}{\sigma}\right)} = F_Z\left(\frac{t-\mu}{\sigma}\right) = \boxed{\Phi\left(\frac{t-\mu}{\sigma}\right)}$$

PER CALCOLARE LA F.d.R. PER QUALSIASI GAUSSIANA SI USERÀ LA F.d.R. DELLA GAUSSIANA STANDARD.

$$f_X(t) = \frac{1}{\sigma\sqrt{2\pi}} \cdot e^{-\frac{(t-\mu)^2}{2\sigma^2}}$$ ← DENSITÀ DI UNA GAUSSIANA QUALSIASI

$$X \sim N(\mu, \sigma^2)$$ ← LEGGE GAUSSIANA O NORMALE

ESEMPIO (GAUSSIANA)

$$X \sim N(\underset{\mu}{250}, \underset{\sigma^2}{9}) \qquad \sigma = 3$$

$$F_X(t) = P(X \le t) = \Phi\left(\frac{t-\mu}{\sigma}\right)$$

(1) $P(X < 254) =$

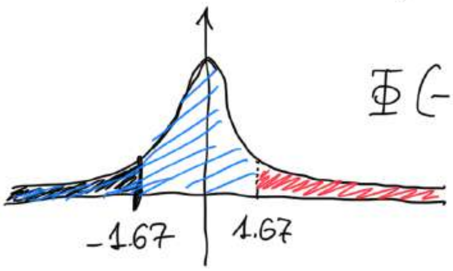
$$P(X \le 254) = \Phi\left(\frac{254-250}{3}\right) = \Phi\left(\frac{4}{3}\right) \simeq$$

$$\simeq \Phi(1.33) \simeq 0.9082 = 90.82\%$$

(2) $P(X < 245) =$

$$P(X \le 245) = \Phi\left(\frac{245-250}{3}\right) = \Phi\left(-\frac{5}{3}\right) \simeq \Phi(-1.67)$$

$$= 1 - \Phi(1.67) = 1 - 0.9525 = 0.0475 = \boxed{4.75\%}$$



$$\Phi(-1.67) = 1 - \Phi(1.67)$$

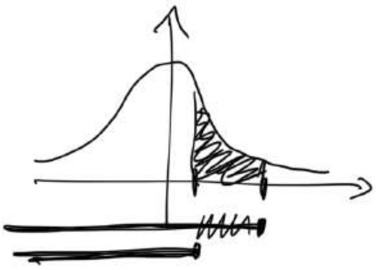
$$\boxed{\Phi(-t) = 1 - \Phi(t)}$$

(3) $P(X > 250) = 1 - P(X \le 250) =$

$$= 1 - \Phi\left(\frac{250-250}{3}\right) = 1 - \Phi(0) = 1 - 0.5 = \boxed{0.5}$$

(4) $P(247 \le X \le 253) = P(X \le 253) - P(X \le 247) =$

$$= \Phi\left(\frac{253-250}{3}\right) - \Phi\left(\frac{247-250}{3}\right)$$



$$= \Phi(1) - \Phi(-1) =$$

$$= \Phi(1) - [1 - \Phi(1)] =$$

$$= 2\Phi(1) - 1 =$$

$$= 2(0.8413) - 1 = 0.6826 =$$

$$= 68.26\%$$

(5) $P(X \le t) = 0.9$

$$\Phi\left(\frac{t-250}{3}\right) = 0.9$$

$$\Rightarrow \frac{t-250}{3} = \boxed{1.28} \Rightarrow t = 1.28 \cdot 3 + 250$$

$$= \boxed{253.84}$$