Esercizi
Logica Proposizionale

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2007

Understanding Implication

$$P \rightarrow Q$$

What can we say when P is false?

Understanding Implication

$$P \rightarrow Q$$

Assume P represents “has libretto” and Q represents “is student”.
“If one has a libretto, (s)he is a student.”

Understanding Implication

$$P \rightarrow Q$$

P	Q	$P \rightarrow Q$
"doesn't have libretto"	"is no student"	OK!
"has libretto"	"is no student"	NO!
"doesn't have libretto"	"is student"	OK!
"has libretto"	"is student"	OK!

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Understanding Implication

Exercise: Which formula represents “Students are exactly those who have a libretto?”

P	Q	$P \,?\, Q$
“doesn’t have libretto”	“is no student”	OK!
“has libretto”	“is no student”	NO!
“doesn’t have libretto”	“is student”	NO!
“has libretto”	“is student”	OK!

Understanding Implication

Exercise: Which formula represents “Students are exactly those who have a libretto?”

P	Q	$P\ ?\ Q$
“doesn’t have libretto”	“is no student”	OK!
“has libretto”	“is no student”	NO!
“doesn’t have libretto”	“is student”	NO!
“has libretto”	“is student”	OK!

Eliminate Parentheses

Ex. 1.1 from “Logica a Informatica”:

1. $((A \wedge B) \rightarrow (\neg C))$
2. $(A \rightarrow (B \rightarrow (\neg C)))$
3. $((A \wedge B) \vee (C \rightarrow C))$
4. $(\neg(A \vee ((\neg B) \rightarrow C)))$
5. $(A \rightarrow (B \vee (C \rightarrow D)))$
6. $(\neg((\neg(\neg(\neg A))) \wedge \bot))$
7. $(A \rightarrow (B \wedge ((\neg C) \vee D)))$

Tautologies, Contradictions

Ex. 1.3 from “Logica a Informatica”: Decide whether the following formulas are tautologies or contradictions:

1. $(A \rightarrow (B \rightarrow C)) \rightarrow ((A \rightarrow B) \rightarrow (A \rightarrow C))$
2. $\neg(A \rightarrow \neg A)$
3. $A \vee \neg A$
4. $\perp \rightarrow A$
5. $\neg A \rightarrow (A \rightarrow B)$
6. $(A \wedge B) \wedge (\neg B \vee C)$
7. $A \vee B \rightarrow A \wedge B$
8. $(A \rightarrow C) \rightarrow ((B \rightarrow C) \rightarrow (A \vee B \rightarrow C))$
9. $(A \rightarrow B) \rightarrow ((B \rightarrow \neg C) \rightarrow \neg A)$

Which of these formulas are satisfiable?

Tautologies, Contradictions

Ex. 1.3 from “Logica a Informatica”: Decide whether the following formulas are tautologies or contradictions:

1. $(A \to (B \to C)) \to ((A \to B) \to (A \to C))$
2. $\neg(A \to \neg A)$
3. $A \vee \neg A$
4. $\bot \to A$
5. $\neg A \to (A \to B)$
6. $(A \wedge B) \wedge (\neg B \vee C)$
7. $A \vee B \to A \wedge B$
8. $(A \to C) \to ((B \to C) \to (A \vee B \to C))$
9. $(A \to B) \to ((B \to \neg C) \to \neg A)$

Which of these formulas are satisfiable?

Tautologies, Contradictions

Similar to Ex. 1.4 from “Logica a Informatica”: Decide whether the following formula is satisfiable

$$(A_1 \vee A_2) \wedge (\neg A_2 \vee \neg A_3) \wedge (A_3 \vee A_4) \wedge (\neg A_4 \vee A_5)$$

Equivalence, Consequence

Ex. 1.8 from “Logica a Informatica”: Prove that

1. $\bot \vee B \equiv B$
2. $\neg\bot \wedge B \equiv B$
3. $A \models A$
4. $A \models B$ and $B \models C$ implies $A \models C$
5. $\models A \rightarrow B$ implies $A \wedge B \equiv A$ and $A \vee B \equiv B$
6. $\models A$ implies $A \wedge B \equiv B$
7. $\models A$ implies $\neg A \vee B \equiv B$
8. If $A \models B$ and $A \models \neg B$ then $\models \neg A$
9. If $A \models C$ and $B \models C$ then $A \vee B \models C$

What are A, B and C?

Equivalence, Consequence

Ex. 1.9 from “Logica a Informatica”: Check whether

1. If $A \models B$ then $\neg A \models \neg B$
2. If $A \models B$ and $A \wedge B \models C$ then $A \models C$
3. If $A \vee B \models A \wedge B$ then $A \equiv B$

Transform to equivalent formula in CNF

Ex. 1.13 from "Logica a Informatica": Find equivalent formulas in CNF for

1. $(A \rightarrow B) \rightarrow (B \rightarrow \neg C)$
2. $\neg(A \rightarrow (B \rightarrow \neg C)) \wedge D$
3. $\neg(A \wedge B \wedge (C \rightarrow D))$
4. $\neg(A \leftrightarrow B)$

Transform to equivalent formula in DNF

Ex. 1.13 from “Logica a Informatica”: Find equivalent formulas in DNF for

1. $(A \rightarrow B) \rightarrow (B \rightarrow \neg C)$
2. $\neg(A \rightarrow (B \rightarrow \neg C)) \wedge D$
3. $\neg(A \wedge B \wedge (C \rightarrow D))$
4. $\neg(A \leftrightarrow B)$

Find the Formula!

Similar to Ex. 1.10 from “Logica a Informatica”: Find f such that

A	B	f
0	0	1
0	1	1
1	0	0
1	1	0

Using only $\rightarrow$ and $\perp$?

Find the Formula!

Similar to Ex. 1.10 from “Logica a Informatica”: Find f such that

A	B	f
0	0	1
0	1	1
1	0	0
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Using only $\rightarrow$ and $\perp$?

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Using only $\vee$ and $\neg$?

Find the Formula!

Similar to Ex. 1.13 from “Logica a Informatica”: Find f such that

A	B	f
0	0	1
0	1	0
1	0	0
1	1	0

Using only $\vee$ and $\neg$?

Find the Formula in CNF and DNF!

Ex. 1.17 from “Logica a Informatica”: Find an f (one in CNF, one in DNF) such that

A	B	C	f
0	0	0	1
0	0	1	0
0	1	0	1
0	1	1	0
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	0

Modelling Dinner

Model the following dinner constraints:

- Available dishes:
  - Farfalle al salmone
  - Risotto agli asparagi
  - Tagliatelle ai funghi
  - Filetto di manzo
  - Spigola grigliata
  - Trancia di pesce spada
- We can choose white or red wine.
- We must choose exactly one primo and one secondo.
- Do not eat fish after mushrooms.
- Choose white wine if fish is involved.

Write a formula such that its models correspond to admissible dinner choices.

Reduction to SAT

Reformulate the following questions such that they can be decided using a SAT algorithm:

1. Is $(P \vee (\neg P \rightarrow Q)) \leftrightarrow (P \vee Q)$ valid?
2. Does $P \rightarrow Q$ follow from $\neg Q \rightarrow \neg P$?
3. Is $P \leftrightarrow Q \wedge P$ a contradiction?
4. Is $P \leftrightarrow P \vee \perp$ a tautology?