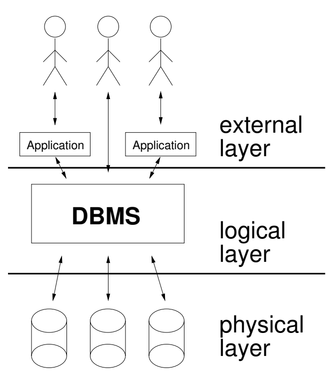
Contents

1 Relational Databases

1.1 Relational Model

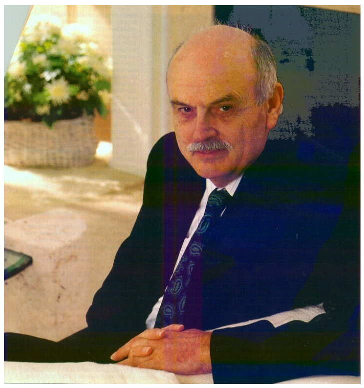
Three Layer Model



Three Layer Model

- *External Layer* How external users view the database.
- *Logical/Conceptual Layer* Logical, holistic view of the database.
- *Physical/Internal Layer* Organisation on the physical media.

Relational Model – Codd 1970


Edgar Frank Codd (1923–2003)

Relations

- Schema:
  - Domain (denumerable set)
  - Attributes (denumerable set)
  - Relations (subset of attributes)
- Instances:
  - Relation instances: Sets of tuples.
  - Each tuple is a function from the relation's attributes to domain elements.
  - Database instance: Collection of relation instances.

Relations: Example

$$A = \{X, Y\}, D = \{a, b, c, d\}$$
$$R = \{X, Y\}, S = \{Y\}$$

$$I(R) = \{t_1, t_2\}$$
$$t_1(X) = a, t_1(Y) = b, t_2(X) = c, t_2(Y) = d$$
$$I(S) = \{t_3\}, t_3(Y) = b$$

$$I(R) = \{\langle a, b\rangle, \langle c, d\rangle\}, I(S) = \{\langle b\rangle\}$$

Relations: Example

R	X	Y
	a	b
	c	d

S	Y
	d

1.2 Relational Algebra

Relational Algebra

Basic Operators:

- σ Selection
- π Projection
- $\times$ Cartesian Product
- $\cup$ Union
- $-$ Difference

Definable using Basic Operators:

- $\bowtie$ Join [$R \bowtie S = \sigma_F(R \times S)$]
- $\ltimes$ Semijoin [$R \bowtie S = \pi_{Schema(R)}(R \bowtie S)$]
- $\cap$ Intersection

Relational Algebra Example

$R \times S$	X_R	Y_R	Y_S
	a	b	d
	c	d	d

$\sigma_{2=3}(R \times S)$	X_R	Y_R	Y_S
	c	d	d

$\pi_{1,2}(\sigma_{2=3}(R \times S))$	X	Y
	c	d

1.3 Relational Model – Logical View

Relations – Logical View

- Schema:
  - Domain – Constant symbols (denumerable set)
  - Relations – Predicate symbols (attributes are not explicitly named)
  - Attributes – implicit by predicate arity
- Instances:
  - Relation instances: Subset of ground instances for relation predicate.
  - Database instance: Subset of Herbrand Base.

Relations: Example

$$D = \{a, b, c, d\}$$
$$R/2, S/1$$
$$I(R) = \{R(a, b), R(c, d)\}, I(S) = \{S(d)\}$$
$$I = \{R(a, b), R(c, d), S(d)\}$$

2 Relational Calculus

Relational Calculus

- Based on First-Order Logic
- Atomic formulas $r(X_1, \ldots, X_n)$
- Comparison formulas $X = 2$ or $X = Y$ (pre-interpreted predicate)
- Composed formulas using $\neg$, $\wedge$, $\exists$
- $\rightarrow$, $\leftrightarrow$, $\vee$, $\forall$ added as "syntactic sugar"

Relational Calculus

- Relational Algebra expressions represent relation instances
- In Relational Calculus: $\{e_1, \ldots, e_n \mid \phi\}$
  - ϕ is a Relational Calculus formula
  - $e_1, \ldots, e_n$: terms containing exactly the free variables of ϕ
- Collect all substitutions for free variables such that ϕ is true in the interpretation formed by the database.
- The defined relation is obtained by applying all of these substitutions to $e_1, \ldots, e_n$.

Relational Calculus Examples

$$\{X, Y, Z \mid R(X, Y) \wedge S(Z)\} = \{T(a, b, d), T(c, d, d)\} = R \times S$$

$$\{X, Y, Y \mid R(X, Y) \wedge S(Y)\} = \{T(c, d, d)\} = \sigma_{2=3}(R \times S)$$

$$\{X, Y \mid R(X, Y) \wedge S(Y)\} = \{T(c, d)\} = \pi_{1,2}(\sigma_{2=3}(R \times S))$$

Algebra as Calculus

- σ_S r $\quad \{X_1, \ldots, X_n \mid r(X_1, \ldots, X_n) \wedge S\}$
- π_i r $\quad \{X_i \mid \exists X_1, \ldots, X_{i-1}, X_{i+1}, \ldots, X_n : r(X_1, \ldots, X_n)\}$
- r $\times$ s $\quad \{X_1, \ldots, X_n, Y_1, \ldots, Y_m \mid r(X_1, \ldots, X_n) \wedge s(Y_1, \ldots, Y_m)\}$
- r $\cup$ s $\quad \{X_1, \ldots, X_n \mid r(X_1, \ldots, X_n) \vee s(X_1, \ldots, X_n)\}$
- r $-$ s $\quad \{X_1, \ldots, X_n \mid r(X_1, \ldots, X_n) \wedge \neg s(X_1, \ldots, X_n)\}$

3 Domain Independence

3.1 Domain Dependent Queries

Calculus: More then Algebra
Problematic expressions:

$$\{X \mid \neg R(a, X)\}$$
$$\{X, Y \mid R(a, X) \vee R(Y, b)\}$$
$$\{X \mid \forall Y : R(X, Y)\}$$

Calculus: More then Algebra
Using the *domain* of the database:

- $\{X \mid \neg R(a, X)\}$
  - all constants c of the domain such that (a, c) is no tuple in R
  - will be infinite if the domain is infinite
- $\{X, Y \mid R(a, X) \vee R(Y, b)\}$
  - if R contains some tuple (a, b), (b, c) for all constants c in the domain
  - will be infinite if the domain is infinite
- $\{X \mid \forall Y : R(X, Y)\}$
  - this will be always empty if the domain is infinite, because relations are finite

Calculus: More then Algebra

Using the *active domain* of the database (only constants appearing in the database and the query):

- $\{X \mid \neg R(a, X)\}$
  - all constants c in the database such that (a, c) is no tuple in R
  - will change if some unrelated constant is added
- $\{X, Y \mid R(a, X) \vee R(Y, b)\}$
  - if R contains some tuple (a, b), (b, c) for all constants c in the database
  - will change if some unrelated constant is added
- $\{X \mid \forall Y : R(X, Y)\}$
  - will unintuitively become empty if an unrelated constant is added

Natural versus Active Domain Semantics

1. *Natural Semantics*: Interpretations from Database Domain
   - pro: Classical First-Order theory
   - contra: Produces infinite relations
   - contra: Quantification over infinite sets
2. *Active Domain Semantics*: Interpretations from Active Domain
   - pro: Always finite
   - contra: Frequently gives unintuitive results
   - contra: Active Domain not always available

3.2 Domain Independent Queries

Domain Independent Queries

Idea: Consider only those queries for which Natural and Active Domain Semantics coincide.

Definition 1. A query in the relational calculus is *domain independent*, if it yields the same answer using the natural (full) domain and the active domain.

Domain Independent Queries

Theorem 2. *Any query of the Relational Algebra can be written as a domain independent query of Relational Calculus, and vice versa.*

Great, let's use only domain independent queries of Relational Calculus.

Domain Independent Queries

Theorem 3. *Deciding whether a query of Relational Calculus is domain independent, is undecidable.*

3.3 Safe Range Queries

Safe Range Queries

Define a syntactically restricted fragment of Relational Calculus queries, which is guaranteed to be domain independent.

1. Transform formula into a normal form (SRNF).
2. Determine range restricted variables of the SRNF formula.
3. Check whether the range restricted variables are exactly the free variables.

SRNF

- Normalize variables: Rename variables, so that each quantifier binds a distinct variable and free and bound variables are different.
- Remove $\forall$: $\forall X : \phi \Rightarrow \neg\exists X : \neg\phi$
- Remove $\rightarrow$: $\phi \rightarrow \psi \Rightarrow \neg\phi \vee \psi$
- Remove $\neg\neg$: $\neg\neg\phi \Rightarrow \phi$
- Push $\neg$: $\neg(\phi \wedge \psi) \Rightarrow (\neg\phi \vee \neg\psi)$
- Push $\neg$: $\neg(\phi \vee \psi) \Rightarrow (\neg\phi \wedge \neg\psi)$

Apply these rules as until none is applicable.

Range Restricted Variables

Intuition: Variables, for which the value is determined by the database.

- Variables in relational atoms are range restricted.
- Variables in equality comparisons with a constant are range restricted.
- Variables in conjunctions are range restricted if they are range restricted in the subformulas.
- Variables in disjunctions are only range restricted if they occur range restricted in both subformulas.
- Variables in negated formulas are never range restricted.
- Variables in existentially quantified subformulas (without the quantified variable) are range restricted if the quantified variable is range restricted in the subformula.

Range Restriction Algorithm

Funtion rr

Input: Formula ϕ in SRNF

Output: Subset of free variables of ϕ or $\perp$

case ϕ of

- $R(t_1, \ldots, t_n)$: $rr(\phi) =$ all variables in $t_1, \ldots, t_n$;
- $X = a$ or $a = X$: $rr(\phi) = \{X\}$;
- $\phi_1 \wedge \phi_2$: $rr(\phi) = rr(\phi_1) \cup rr(\phi_2)$;
- $\phi_1 \wedge X = Y : rr(\phi) = \begin{cases} rr(\phi_1) & \text{if } \{X, Y\} \cap rr(\phi_1) = \emptyset; \\ rr(\phi_1) \cup \{X, Y\} & \text{otherwise;} \end{cases}$
- $\phi_1 \vee \phi_2$: $rr(\phi) = rr(\phi_1) \cap rr(\phi_2)$;
- $\neg\phi_1$: $rr(\phi) = \emptyset$;
- $\exists X : \psi$: if $X \in rr(\psi)$ then $rr(\phi) = rr(\psi) \setminus \{X\}$ else return $\perp$;

Assumption: Set operations with $\perp$ always result in $\perp$.

Safe Range Queries

Definition 4. A Relational Calculus query $\{e_1, \ldots, e_n \mid \phi\}$ is *safe range*, if $rr(SRNF(\phi))$ is equal to the free variables in ϕ.

Theorem 5. *Each safe range query is domain independent.*

Theorem 6. *Any safe range query can be written as query of Relational Algebra, and vice versa.*

3.4 SQL

SQL

- Exists since 1974 (developed by IBM).
- ISO/ANSI standardization 1986/87.
- First extension 1989.
- Second extension 1992 (SQL-92/SQL-2).
- Last (up to now) extension 1999/2000 (SQL-99/SQL-3)
- SQL combines query and manipulation languages.

SQL
SELECT P FROM C WHERE S

- P — Projections
- C — Cartesian Product
- S — Selections

SQL
Union:
SELECT ... UNION SELECT ...
Nesting:
SELECT P FROM C WHERE A [NOT] IN (SELECT ...)

SQL

Theorem 7. *The query portion of SQL-92 basically corresponds to Relational Algebra, and hence to safe range Relational Calculus.*

4 Datalog

4.1 Motivation

Recursion

- Some simple problems cannot be represented in relational calculus.
- Example: Reachability on deterministic graphs.
- Prototypical for LOGSPACE!
- Holds also for relational algebra, SQL-92 etc.

Transitive Closure
Key notion: Transitive Closure

Definition 8. Given graph $G = \langle V, E\rangle$, $E \subseteq V \times V$, and $a, b \in V$, the *transitive closure* $TC(G) \subseteq V \times V$ is:

$$\begin{aligned} TC(G) = \; & \{(x,y) \mid (x,y) \in E\} \\ & \cup\{(x,y) \mid (x,z) \in TC(G) \wedge (z,y) \in TC(G)\} \end{aligned}$$

Note: $TC(G)$ appears in its own definition.
In relational calculus we cannot refer to what we define.

- Idea: Use Horn clauses for named definitions.
- It is then possible to write definitions using the concept being defined.
- Positive Datalog

4.2 Syntax

Language Elements

- Set of extensional predicate symbols $\mathbf{PS_{EDB}}$
- Set of intensional predicate symbols $\mathbf{PS_{IDB}}$
- $\mathbf{PS_{EDB}} \cap \mathbf{PS_{IDB}} = \emptyset$
- Each predicate symbol has an associated arity $ar : \mathbf{PS_{EDB}} \cup \mathbf{PS_{IDB}} \rightarrow N_0$
- Set of constant symbols **CS**
- Set of variable symbols **VS**

Syntax

A Datalog rule is of the form:

$$r_1(t_{1_1}, \ldots, t_{n_1}) \leftarrow r_2(t_{1_2}, \ldots, t_{n_2}), \ldots, r_m(t_{1_m}, \ldots, t_{n_m}).$$

- $m \geq 1$
- $r_1 \in \mathbf{PS_{IDB}}$
- $r_2, \ldots, r_m \in \mathbf{PS_{EDB}} \cup \mathbf{PS_{IDB}}$
- $t_{1_1}, \ldots, t_{n_m} \in \mathbf{CS} \cup \mathbf{VS}$
- $\forall i\ 1 \leq i \leq m : ar(r_i) = n_i$
- $((t_{1_1} \cup \ldots \cup t_{n_1}) \cap \mathbf{VS}) \subseteq ((t_{1_2} \cup \ldots \cup t_{n_m}) \cap \mathbf{VS})$

Syntax

$$r_1(t_{1_1}, \ldots, t_{n_1}) \leftarrow r_2(t_{1_2}, \ldots, t_{n_2}), \ldots, r_m(t_{1_m}, \ldots, t_{n_m}).$$

- $H(r) = \{r_1(t_{1_1}, \ldots, t_{n_1})\}$
- $B(r) = \{r_2(t_{1_2}, \ldots, t_{n_2}), \ldots, r_m(t_{1_m}, \ldots, t_{n_m})\}$
- $V(r) = \{t_{1_1}, \ldots, t_{n_m}\} \cap \mathbf{VS}$
- $C(r) = \{t_{1_1}, \ldots, t_{n_m}\} \cap \mathbf{CS}$

- $H(r)$ is the *head* of r.
- $B(r)$ is the *body* of r.
- A *Datalog program* is a set of rules.

4.3 Semantics

Semantics

Intuitively: For each rule r, whenever $B(r)$ is true, $H(r)$ should also be true. $B(r) = \emptyset$ is considered to be true.

Different ways for defining the semantics:

- *model theory*
- *fixpoint theory*
- *proof theory*

4.3.1 Model Theory

Model Theory

Definition 9 (Herbrand Universe)**.**

$$\mathbf{HU}(\mathcal{P}) = \bigcup_{r \in \mathcal{P}} C(r)$$

Definition 10 (Herbrand Base)**.**

$$\mathbf{HB}(\mathcal{P}) = \{r(t_1, \ldots, t_n) \mid r \in \mathbf{PS_{EDB}} \cup \mathbf{PS_{IDB}},\\ t_1, \ldots, t_n \in \mathbf{HU}(\mathcal{P}), ar(r) = n\}$$

- $\mathbf{HU}(\mathcal{P})$: Constants of the program (active domain!)
- $\mathbf{HB}(\mathcal{P})$: Ground atoms constructible from $\mathbf{HU}(\mathcal{P})$

Example: Herbrand Base

Example 11.

$$\begin{aligned}\mathcal{P}_r = \{\ & \mathtt{arc(a,b).}\\ & \mathtt{arc(b,c).}\\ & \mathtt{reachable(a).}\\ & \mathtt{reachable(Y)} \leftarrow \mathtt{arc(X,Y), reachable(X).}\ \}\end{aligned}$$

$$\begin{aligned}\mathbf{HU}(\mathcal{P}_r) = & \ \{\mathtt{a,b,c}\}\\ \mathbf{HB}(\mathcal{P}_r) = & \ \{\mathtt{arc(a,a), arc(a,b), arc(a,c),}\\ & \ \mathtt{arc(b,a), arc(b,b), arc(b,c),}\\ & \ \mathtt{arc(c,a), arc(c,b), arc(c,c),}\\ & \ \mathtt{reachable(a), reachable(b), reachable(c)}\}\end{aligned}$$

Instantiation

Definition 12. *Valuation* $v_{\mathcal{P}}(r)$ of a rule r: Set of all substitutions $V(r) \to \mathbf{HU}(\mathcal{P})$

Definition 13 (Instantiation of a rule r)**.** $Ground_{\mathcal{P}}(r) = \bigcup_{v \in v_{\mathcal{P}}(r)} v(r)$

Definition 14 (Instantiation of a program $\mathcal{P}$)**.** $Ground(\mathcal{P}) = \bigcup_{r \in \mathcal{P}} Ground_{\mathcal{P}}(r)$

Example: Instantiation

Example 15.
$$\mathcal{P}_r = \{ \quad \texttt{arc(a,b). arc(b,c). reachable(a).} \\ \texttt{reachable(Y)} \leftarrow \texttt{arc(X,Y), reachable(X).} \}$$

$$\begin{aligned} Ground(\mathcal{P}_r) = \{ & \texttt{arc(a,b). arc(b,c). reachable(a).} \\ & \texttt{reachable(a)} \leftarrow \texttt{arc(a,a), reachable(a).} \\ & \texttt{reachable(b)} \leftarrow \texttt{arc(a,b), reachable(a).} \\ & \texttt{reachable(c)} \leftarrow \texttt{arc(a,c), reachable(a).} \\ & \texttt{reachable(a)} \leftarrow \texttt{arc(b,a), reachable(b).} \\ & \texttt{reachable(b)} \leftarrow \texttt{arc(b,b), reachable(b).} \\ & \texttt{reachable(c)} \leftarrow \texttt{arc(b,c), reachable(b).} \\ & \texttt{reachable(a)} \leftarrow \texttt{arc(c,a), reachable(c).} \\ & \texttt{reachable(b)} \leftarrow \texttt{arc(c,b), reachable(c).} \\ & \texttt{reachable(c)} \leftarrow \texttt{arc(c,c), reachable(c).} \} \end{aligned}$$

Herbrand Models

Definition 16 ((Herbrand-) Interpretations I for $\mathcal{P}$)**.** $I \subseteq \mathbf{HB}(\mathcal{P})$

Definition 17 ((Herbrand-) Models for $\mathcal{P}$)**.** $M \subseteq \mathbf{HB}(\mathcal{P})$ such that $\forall r \in Ground(\mathcal{P})$: $(H(r) \subseteq M) \vee (B(r) \not\subseteq M)$

"If the body is true, the head must be true."

Definition 18 ((Herbrand-) Models for $\mathcal{P}$)**.** $M \subseteq \mathbf{HB}(\mathcal{P})$ such that $\forall r \in Ground(\mathcal{P})$: $(B(r) \subseteq M) \to (H(r) \subseteq M)$

Example: Herbrand Models

Example 19.
$$\mathcal{P}_r = \{ \quad \texttt{arc(a,b). arc(b,c). reachable(a).} \\ \texttt{reachable(Y)} \leftarrow \texttt{arc(X,Y), reachable(X).} \}$$

$$M_1 = \{ \quad \texttt{arc(a,b), arc(b,c),} \\ \texttt{reachable(a), reachable(b), reachable(c)} \}$$

$M_2 = \mathbf{HB}(\mathcal{P}_r)$

All $M : M_1 \subseteq M \subseteq M_2$ are models and only these.

Minimal Models

Theorem 20. $\mathbf{HB}(\mathcal{P})$ *is always a model for any Datalog program* $\mathcal{P}$.

Theorem 21. *Each Datalog program* $\mathcal{P}$ *has a unique subset minimal model* $MM(\mathcal{P})$.

Definition 22. The semantics of a Datalog program $\mathcal{P}$ is given by $MM(\mathcal{P})$

Note: Each element of $MM(\mathcal{P})$ is a logical consequence of $\mathcal{P}$.

4.3.2 Fixpoint Theory

Concept: Operator

"If we assume that all atoms in I are true, which other atoms must be true in order to satisfy the program?"

- Start with $I = \emptyset$ (nothing is true).
- Define operator $\mathbf{T}_{\mathcal{P}}$.
- Apply $\mathbf{T}_{\mathcal{P}}$, until there are no further additions.
- The obtained result (fixpoint) defines the semantics.

Immediate Consequences

Definition 23 (Operator $\mathbf{T}_{\mathcal{P}}$ for Datalog program $\mathcal{P}$)**.** Given an interpretation I,

$$\mathbf{T}_{\mathcal{P}}(I) = \{h \mid r \in Ground(\mathcal{P}), B(r) \subseteq I, h \in H(r)\}$$

- $\mathbf{T}_{\mathcal{P}}(I)$ extends I, such that unsatisfied rules (w.r.t. I) become satisfied.
- Other rules may become unsatisfied w.r.t. $\mathbf{T}_{\mathcal{P}}(I)$.
- $\Rightarrow$ Iterative application.

Example: Immediate Consequences

Example 24. $\mathcal{P}_r = \{ \quad \texttt{arc(a,b). arc(b,c). reachable(a).}$
$\texttt{reachable(Y)} \leftarrow \texttt{arc(X,Y), reachable(X).} \}$

1. $\mathbf{T}_{\mathcal{P}_r}(\emptyset) = \{\texttt{arc(a,b)},\ \texttt{arc(b,c)},\ \texttt{reachable(a)}\}$
2. $\mathbf{T}_{\mathcal{P}_r}(\mathbf{T}_{\mathcal{P}_r}(\emptyset)) = \mathbf{T}_{\mathcal{P}_r}(\emptyset) \cup \{\texttt{reachable(b)}\}$
3. $\mathbf{T}_{\mathcal{P}_r}(\mathbf{T}_{\mathcal{P}_r}(\mathbf{T}_{\mathcal{P}_r}(\emptyset))) = \mathbf{T}_{\mathcal{P}_r}(\mathbf{T}_{\mathcal{P}_r}(\emptyset)) \cup \{\texttt{reachable(c)}\}$
4. $\mathbf{T}_{\mathcal{P}_r}(\mathbf{T}_{\mathcal{P}_r}(\mathbf{T}_{\mathcal{P}_r}(\mathbf{T}_{\mathcal{P}_r}(\emptyset)))) = \mathbf{T}_{\mathcal{P}_r}(\mathbf{T}_{\mathcal{P}_r}(\mathbf{T}_{\mathcal{P}_r}(\emptyset)))$
5. $\{\texttt{arc(a,b)},\ \texttt{arc(b,c)},\ \texttt{reachable(a)},\ \texttt{reachable(b)},\ \texttt{reachable(c)}\}$

Properties of $\mathbf{T}_{\mathcal{P}}$

Lattice: $V = (P(\mathbf{HB}(\mathcal{P})), \subseteq)$[0.5cm]
$\forall X \subseteq V : \exists inf(X) \wedge \exists sup(X)$[0.5cm]
$inf(V) = \emptyset$, $sup(V) = \mathbf{HB}(\mathcal{P})$[0.5cm]
Monotony: $X \subseteq Y \rightarrow \mathbf{T}_{\mathcal{P}}(X) \subseteq \mathbf{T}_{\mathcal{P}}(Y)$[0.5cm]
Continuity: $\forall X \subseteq V : \mathbf{T}_{\mathcal{P}}(sup(X)) = sup(\mathbf{T}_{\mathcal{P}}(X))$

Tarski, Kleene

Bronisław Knaster Alfred Tarski Stephen Kleene (1893–1990)
(1902–1983) (1909–1994)

Existence of Fixpoints

Theorem 25. $\mathbf{T}_{\mathcal{P}}$ *is monotone und continuous on the lattice of interpretations and subset relations.*

Theorem 26 (Knaster-Tarski)**.** *For monotone operators on lattices a least fixpoint exists, and it is* $inf(\{X \mid \mathbf{T}_{\mathcal{P}}(X) \subseteq X\})$

Construction of Fixpoints

Theorem 27 (Kleene)**.** *For continuous operators on lattices the least fixpoint can be computed by iteration starting from the infimum.* $\mathbf{T}^{\omega}_{\mathcal{P}} = sup(\{\mathbf{T}^{i}_{\mathcal{P}} \mid i \geq 0\})$, $\mathbf{T}^{0}_{\mathcal{P}} = inf(V)$, $\mathbf{T}^{i}_{\mathcal{P}} = \mathbf{T}_{\mathcal{P}}(\mathbf{T}^{i-1}_{\mathcal{P}})$

Corollary 28. *Our lattice is finite, therefore the least fixpoint of* $\mathbf{T}_{\mathcal{P}}$ *can be computed by a finite number of itrations starting from* $\emptyset$.

$\mathbf{T}^{\omega}_{\mathcal{P}}$ – Minimal Model

Theorem 29. *For all Datalog programs* $\mathcal{P}$*, we can show* $\mathbf{T}^{\omega}_{\mathcal{P}} = MM(\mathcal{P})$.

Note: All consequences of a program can be computed by iteration over the immediate consequences.

4.3.3 Proof Theory

Reminder: Horn and Goal Clauses, SLD Resolution

- A *Horn clause* is a clause containing at most one positive literal.
- A *Goal clause* is a clause containing no positive literal.
- *SLD Resolution*: Linear resolution, where at each step only goal clauses and (instances of) input clauses are used.

Theorem 30. *SLD resolution is refutation complete for Horn clauses.*

SLD Resolution for Datalog

- We can view each rule as a Horn clause.
- So SLD Resolution can be applied.
- Unification is simpler for Datalog because of absence of function symbols.

Definition 31 (SLD Resolution Semantics)**.** Let $SLD(\mathcal{P})$ denote the set of ground atoms, for which an SLD refutation w.r.t. $\mathcal{P}$ exists.

Equivalence

Theorem 32. *For all Datalog programs $\mathcal{P}$, we can show $SLD(\mathcal{P}) = \mathbf{T}_{\mathcal{P}}^{\omega} = MM(\mathcal{P})$.*

SLD Tree

Top-down and **bottom-up** views:[0.5cm]

1. *Top-down*: Start at the root.
2. *Bottom-up*: Start at leaves.

$\mathbf{T}_{\mathcal{P}}^{\omega}$: Is like SLD bottom-up (on finite branches).

SLD Resolution – Termination

$\texttt{child\_of}(\texttt{charles}, \texttt{francis}).$
$\texttt{child\_of}(\texttt{francis}, \texttt{frida}).$
$\texttt{successor\_of}(\texttt{X}, \texttt{Y}) \leftarrow \texttt{child\_of}(\texttt{X}, \texttt{Y}).$
$\texttt{successor\_of}(\texttt{X}, \texttt{Y}) \leftarrow \texttt{child\_of}(\texttt{X}, \texttt{Z}), \texttt{successor\_of}(\texttt{Z}, \texttt{Y}).$
$\leftarrow \texttt{successor\_of}(\texttt{charles}, \texttt{X}).$

SLD Resolution – Termination

$\texttt{child\_of}(\texttt{charles}, \texttt{francis}).$
$\texttt{child\_of}(\texttt{francis}, \texttt{frida}).$
$\texttt{successor\_of}(\texttt{X}, \texttt{Y}) \leftarrow \texttt{child\_of}(\texttt{X}, \texttt{Y}).$
$\texttt{successor\_of}(\texttt{X}, \texttt{Y}) \leftarrow \texttt{successor\_of}(\texttt{X}, \texttt{Z}), \texttt{child\_of}(\texttt{Z}, \texttt{Y}).$
$\leftarrow \texttt{successor\_of}(\texttt{charles}, \texttt{X}).$

SLD Resolution – Termination

$\texttt{child\_of}(\texttt{charles}, \texttt{francis}).$
$\texttt{child\_of}(\texttt{francis}, \texttt{frida}).$
$\texttt{successor\_of}(\texttt{X}, \texttt{Y}) \leftarrow \texttt{child\_of}(\texttt{X}, \texttt{Y}).$
$\texttt{successor\_of}(\texttt{X}, \texttt{Y}) \leftarrow \texttt{successor\_of}(\texttt{X}, \texttt{Z}), \texttt{successor\_of}(\texttt{Z}, \texttt{Y}).$
$\leftarrow \texttt{successor\_of}(\texttt{charles}, \texttt{X}).$

4.4 Computation

Simple Algorithms

From the semantic definitions, we can produce simple algorithms:

- *Model Theory*: Enumerate all subsets of $\mathbf{HB}(\mathcal{P})$, test whether they are models and take the minimal one.
- *Fixpoint Theory*: Extend $\emptyset$ by applying $\mathbf{T}_{\mathcal{P}}$ until a fixpoint is reached.
- *Proof Theory*: Use SLD Resolution bottom-up.

Simple Algorithms 2

Also for query answering we can find simple algorithms:

- Straightforward: Compute model and test whether query is true.
- Better: Use SLD resolution top-down.

Termination?