

ON ITERATIVE METHODS FOR QUASI-NONEXPANSIVE MAPPINGS

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Abstract

Many of the most important nonlinear problems of applied mathematics reduce to finding solutions of nonlinear functional equations (nonlinear integral equations, boundary value problems for nonlinear ordinary equations or partial differential equations, the existence of periodic solutions of nonlinear partial differential equations). They can be formulated, in terms of finding the fixed points of a given nonlinear mapping T defined on a nonempty set of a Banach space X into itself, by the following equation

$$x = Tx, \quad x \in X. \quad (0.1)$$

The solutions of the equation (0.1) are called *fixed points of T* . The aim of this Thesis is to solve some proposed problems, related to approximation of fixed points of a nonlinear mapping or common fixed points of families of mappings and approximation of solutions of variational inequalities, illustrated in an unified treatment and collection of the main ideas, concepts and important results on our iterative methods.

We will present our contribution to the following proposed problems.

Problem 1:

*We do not know whether a strong convergence Theorem of Halpern's type for nonspreading mappings holds or not, (see Y.Kurokawa, W.Takahashi, Weak and strong convergence theorems for nonspreading mappings in Hilbert spaces, *Nonlinear Anal.* 73, no. 6, (2010), 1562-1568).*

We give a partial answer in F. Cianciaruso, G. Marino, A. Rugiano, B. Scardamaglia, *On Strong convergence of Halpern's method using averaged type mappings*, J. Appl. Math., (2014), Art. ID 473243, 11 pages.

In Chapter 2, we discuss our results.

Problem 2: *One can ask if a strong convergence Theorem for a viscosity type method to approximate common fixed points of two mappings holds or not.*

We present a partial answer in F. Cianciaruso, G. Marino, A. Rugiano, B. Scardamaglia, *On strong convergence of viscosity type method using averaged type mappings*, Journal of Nonlinear and Convex Analysis no. 8, vol. 16, (2015), 1619-1640. For a deep understanding, we discuss our results in Chapter 3.

Problem 3: *It is interesting to understand for which classes of mappings a strong convergence Theorem for a Halpern's type method holds or not, without the introduction of averaged type mappings.*

We discuss a partial answer in J. Garcia Falset, E. LLorens Fuster, G. Marino, A. Rugiano, *On strong convergence of Halpern's method for quasi-nonexpansive mappings in Hilbert Spaces*, in print in Mathematical Modelling and Analysis.

We show our results in Chapter 4.

Problem 4: *One can ask if a strong convergence Theorem for the iterative scheme introduced by Iemoto and Takahashi (see S.Iemoto, W.Takahashi, Approximating common fixed points of nonexpansive mappings and nonspreading mappings in a Hilbert space *Nonlinear Anal.* 71, no. 12, (2009), 2082-2089), holds or not.*

We illustrate a partial answer in A. Rugiano, B. Scardamaglia, S. Wang, *Hybrid iterative algorithms for a finite family of nonexpansive mappings and for a countable family of nonspreading mappings in Hilbert spaces*, to appear in *Journal of Nonlinear and Convex Analysis*.

We analyze our results in Chapter 5.

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