Abstract

The research of this thesis is part of the study of some problems in extremal combinatorial set theory, which are related to some aspects of the analytic number theory. We placed these problems in the lattice theory and using classic tools we e have obtained some results, which are related to the well-known Manickham-Miklós-Singhi-conjecture (1988). In particular, we found the maximum and the minimum values related to a class of multi-sets of real numbers with non-negative sums. Moreover we builded a Boolean map for each intermediate value between the maximum and the minimum. This Boolean map represents the abstract form of each multi-sets.

However, the lattice theory provided a partial solution. Afterwards, with different approach, we solved a generalization of a problem of Manickam e Miklós.

The last goal was to adopt analytical methods to find some estimates, which come from property of graded Posets.

Any ranked poset whose largest antichain is not bigger than its maximum level is said to possess the Sperner property, hence the Sperner property is equivalently the property that some rank level is a maximum antichain. Many tools have been developed by researchers to determine whether a poset has the property of Sperner, instead there are considerably fewer techniques for establishing if a poset has not the Sperner property. In particular, we studied a quotient of lattice of partition of the set $[n] = \{1, 2, \ldots, n\}$ and we estimate how much it is 'close' to be a Sperner poset.