Outflanking quantum complementarity: detection of incompatible properties in double-slit experiments.

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Standard Quantum Theory [1] denies the possibility of measuring simultaneously (i.e. on the same individual specimen of the physical system) non-commuting observables; the best example describing this physical situation is the double-slit experiment: properties such as "the particle passes through the first (second) slit" and "the particle hits the final screen in a point within the interval Δ" cannot be measured together, since they are mathematically represented by non-commuting projection operators, E and $F(\Delta)$ respectively. This notwithstanding, several devices are conceived over the years providing *indirect* knowledge of WS property; among them, the experiment presented in [2] provides WS knowledge by exploiting recent advances in quantum optics: instead of measuring WS property, a different property T is measured, correlated with it and compatible with the measurement of the final impact point; this kind of detection is studied from a mathematical point of view in [3] and [4], where the interpretative questions are analyzed. In [5] the problem of detecting three incompatible properties in the framework of double-slit experiment is treated: in some circumstances, inferences about more incompatible properties can be done. This kind of detection is possible, for instance, if besides the position of the centre-of-mass, the system possesses further degrees of freedom; as a consequence, the Hilbert space describing the entire system can be decomposed as $\mathcal{H}_I \otimes \mathcal{H}_{II}$, where $\mathcal{H}_I$ is the Hilbert space used to represent the position observable, and $\mathcal{H}_{II}$ is the Hilbert space used to represent the observable arising from the further degrees of freedom. The detection of Which Slit property E is obtained by measuring an observable represented by a particular projection operator T acting on $\mathcal{H}_{II}$, which is correlated, in the particular quantum state of the system, with which-slit property, so that this last can be inferred from the outcome of T. The possibility of detecting an incompatible property G is provided by the existence of an observable represented by another projection operator Y acting on $\mathcal{H}_{II}$, but which can be measured together with T. A systematic investigation establishes that the existence of such an observable (projection operator) depends on the dimension of space $\mathcal{H}_I$ [5]. In this work we are interested in the research of non-correlated solutions: a wide family of solutions is provided for $dim(\mathcal{H}_I) = 6$ and an ideal experiment realizing such kind of detection is designed. We have to notice that this approach to the problem of inferring the outcomes of more incompatible properties is not the only one. In [6], a procedure is described allowing to make inferences about the three cartesian components of a spin-$\frac{1}{2}$ particle. However, this kind of inferences are of a quite different nature; moreover, coherently with this second method, it is not possible to produces inferences about more than three observables. For this reason, the question whether two mutually incompatible properties, G and L, both incompatible with Which Slit property E, can be detected, together with the measurement of the final impact point (four incompatible properties), is investigated. In particular, we show that such a question has an affirmative answer; as in the previous case, the existence of solutions depends on dimension of space $\mathcal{H}_I$; we find a particular solution for $dim(\mathcal{H}_I) = 10$, nevertheless, in such a case the properties L and G turn out to be correlated.

The content of this thesis is organized in three chapters.

In chapter 1 Von Neumann theory is presented, in a modern form; once introduced the axiomatic basis, several consequences follow; for instance, since the algebraic structure of self-adjoint operators is not commutative, then it is not possible to measure on an individual specimen of the physical system (we shall say *simultaneously*) all the observables but only those corresponding to commutative self-adjoint operators. Hence, in a natural manner, the concept of complementarity is introduced, following the approach proposed by Busch and Lahti [7]. This chapter has an introductory character; its role is to illustrate the notions involved in the thesis and to establish the notation.

In chapter 2 we examine the double-slit experiment, according to the quantum theory; after a detailed mathematical description, we can see that empirical results are perfectly in agreement with the quantum theoretical predictions; moreover, now we are able to solve the interpretative question: WS property cannot be measured together with the final impact point because they are complementary observables; indirect WS knowledge can be obtained if, instead of measuring it, a different property T is measured, compatible with the measurement of the final impact point and correlated with WS property; this is the idea beyond ESW experiment.

In chapter 3 the question of ascertaining together more incompatible properties is investigated, in the framework of the double-slit experiment, by means of the notion of detector. At first, an ideal experiment where a property L, incompatible with WS property, is detected together with the measurement of the final impact point and together with the detection of WS property is presented; however in the situation envisaged (similar to ESW experiment) the two detections turn out to be correlated. Then, a systematic investigation of the problem of ascertaining together WS property, an incompatible one G and the final impact point (three incompatible properties) is carried out. We have to stress the difference between the nature of inferences provided by our method of attaining indirect knowledge with respect to other ones. In particular, we refer to the procedure presented in [6]; since it is inadequate for the detection of more than three observables, we treat the case of four incompatible properties in the same kind of ideal experiment, showing that it has affirmative answer.

References

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