Lectures in Nonlinear Analysis and Differential Equations

Doctoral School in Mathematics and Computer Science
Department of Mathematics and Computer Science, University of Calabria

May 23-27, 2016

General Information

The course is intended for doctoral students and young researchers interested in Nonlinear Analysis and Differential Equations.
The classes will be given in Aula MT11, Department of Mathematics and Computer Science, University of Calabria, Cubo 30B, first floor.
During the same period our University will host three talks given by some participants, these included here for completeness.

Course Lecturers

- **Daria Bugajewska**, Adam Mickiewicz University, Poland, dbw@amu.edu.pl,
  Nonlinear operators in the spaces of functions of bounded variation of various types and their applications.

  The theory of functions of bounded variation was initiated by Jordan in 1881, but still many its problems (even basic ones) remain open. In particular, it should be mentioned here, the questions concerning nonautonomous superposition operators acting in the spaces of functions of bounded variation in the sense of Jordan or in other classical spaces of functions of bounded variation. For example, the issue of finding necessary and sufficient conditions under which the nonautonomous superposition operator maps the space of functions of bounded variation in the sense of Jordan into itself still remains open. In these lectures we are going to present most of all the new results concerning nonlinear operators in the spaces of functions of bounded variation in the sense of Jordan. Moreover we will give some applications of our results in the theory of differential equations and nonlinear integral equations.

  On the other hand, a very interesting generalization of the notion of bounded variation in the sense of Jordan, namely the Λ-bounded variation, was introduced by Waterman in 1972. We are going to present a new notion of Λ-bounded variation, defined on a class of functions which are equal almost everywhere. This new notion extends the variation in the sense of Waterman. In particular, we will discuss the properties of so called "good representatives", that is functions Λ-bounded variation of which is equal to Λ-bounded variation (in our sense) of classes of equivalence, under consideration. Moreover, some geometric and topological properties of spaces of functions of this type will be provided.

- **Petru Jebelean**, West University of Timisoara, Romania, jebelean@math.uvt.ro,
  Singular ϕ-Laplacians - fixed points and variational methods.

  The lectures are organised as follows:

  1. Radial solutions for Minkowski operator;
  2. Some types of critical points - an overview;
  3. Periodic solutions for singular differential operators;
  4. Back to the Minkowski operator;

5. Further developments of the subject.

- **Piotr Kasprzak**, Adam Mickiewicz University, Poland, kasp@amu.edu.pl,
Eigenvalues of nonlinear operators with applications to integral and differential equations.

  In the study of nonlinear differential and integral equations fixed point theorems for compact operators play an important role. Such results often allow not only to prove the existence of a solution to a given problem, but also to obtain some additional information about its properties/behaviour. During the lecture we will discuss certain results concerning the existence of positive invariant directions (or positive eigenvalues) for compact mappings, which are closely connected with the fixed point theory. One of the most classical examples of such results is the Birkhoff–Kellogg theorem. It states that if a compact mapping defined on a boundary of an open and bounded neighbourhood of zero U in an infinite-dimensional normed space satisfies the following condition $\|F(x)\| \geq d > 0$ for all $x \in \partial U$, then F has an invariant direction. However, it turns out that from the point of view of applications, in some cases, it is more convenient to use an extension of the result due to Bikrhoff and Kellogg, that is, the theorem of Leggett and Williams, which not only guarantees the existence of a positive invariant direction of the considered mapping, but also shows that the corresponding eigenvector satisfies an additional condition formulated in terms of a given continuous, positively homogeneous and additive functional. During the lecture we will present this theorem along with its generalizations. Furthermore, we will show how to apply the aforementioned results to nonlinear differential and integral equations. In particular, we will show that under certain assumptions concerning the positivity of the given function f, it is possible to prove an existence of a positive parameter λ such that the ODE $-x''(t) = \lambda f(t, x(t))$ with a certain boundary condition has a positive solution.

Guest Speakers

- **Pasquale Candito**, Mediterranean University of Reggio Calabria, Italy, pasquale.candito@unirc.it,
A coincidence point approach in solving nonlinear problems.

  The aim of this talk is to present a recent existence result for the following coincidence equation

$$Fu = Gu,$$

  where X and Y are two real Banach spaces and $F, G : X \to Y$ are two suitable nonlinear operators which are not necessarily invertible, [1]. Moreover, some of its applications to the study of different nonlinear boundary value problems are shown, [1], [2]. A general overview on this topics can be find in [3].

  References

  1. G. Bonanno, P. Candito, D. Motreanu, *A coincidence point theorem for sequentially continuous mappings*, J. Math. Anal. Appl. **435** (2016), no. 1, 606-615.
  2. P. Candito, R. Livrea, *An existence result for a Neumann problem*, Dyn. Contin. Discrete Impuls. Syst. Ser. A Math. Anal. **22** (2015), no. 6, 481-488.
  3. E. Zeidler, *Nonlinear Functional Analysis and Its Applications*, Vol. II B, Chap. 29. Springer-Verlag, Berlin 1990.

- **Roberto Livrea**, Mediterranean University of Reggio Calabria, Italy, roberto.livrea@unirc.it,
Existence and multiplicity for second order dynamical systems.

  Variational methods are used in order to establish the existence and the multiplicity of nontrivial periodic solutions of a second order dynamical system. The main results are obtained when the potential satisfies different conditions at zero and at infinity.

- **Feliz Minhós**, University of Evora, Portugal, fminhos@uevora.pt,
  Solvability of third order nonlinear coupled systems.

  This talk will be about sufficient conditions for the solvability of the third order three point boundary value problem

$$\begin{cases} -u'''(t) = f(t,\, v(t),\, v'(t)) \\ -v'''(t) = h(t,\, u(t),\, u'(t)) \\ u(0) = u'(0) = 0, u'(1) = \alpha u'(\eta) \\ v(0) = v'(0) = 0, v'(1) = \alpha v'(\eta). \end{cases}$$

  The arguments apply Green's function associated to the linear problem and the Guo–Krasnosel'skiĭ theorem of compression-expansion cones.

  The dependence on the first derivatives is overcome by the construction of an adequate cone and suitable conditions of superlinearity/sublinearity near 0 and $+\infty$.

Course Schedule

MONDAY 23 MAY:

9:00-11:00	Petru Jebelean
	Singular ϕ-Laplacians - fixed points and variational methods (Part 1)
11:00-12:00	Feliz Minhós
	Solvability of third order nonlinear coupled systems

TUESDAY 24 MAY:

9:00-11:00	Daria Bugajewska
	Nonlinear operators in the spaces of functions of bounded variation and applications (Part 1)
11:00-13:00	Piotr Kasprzak
	Eigenvalues of nonlinear operators with applications to integral and DE (Part 1)

WEDNESDAY 25 MAY:

9:00-11:00	Petru Jebelean
	Singular ϕ-Laplacians - fixed points and variational methods (Part 2)
11:00-13:00	Daria Bugajewska
	Nonlinear operators in the spaces of functions of bounded variation and applications (Part 2)

THURSDAY 26 MAY:

9:00-11:00	Piotr Kasprzak
	Eigenvalues of nonlinear operators with applications to integral and DE (Part 2)
11:00-13:00	Petru Jebelean
	Singular ϕ-Laplacians - fixed points and variational methods (Part 3)
15:00-15:30	Pasquale Candito
	A coincidence point approach in solving nonlinear problems
15:30-16:00	Roberto Livrea
	Existence and multiplicity for second order dynamical systems
16:00-18:00	Course Lecturers
	Tutorials

FRIDAY 27 MAY:

9:00-11:00	Daria Bugajewska
	Nonlinear operators in the spaces of functions of bounded variation and applications (Part 3)
11:00-13:00	Piotr Kasprzak
	Eigenvalues of nonlinear operators with applications to integral and DE (Part 3)