Propositional Logic Exercises

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A.Y. 2018/2019

Outline

Understanding: Mode of reasoning (1)

Deduction, induction, or abduction?

1. You are walking on the bridge at Unical for the first time, South to North direction
   - You see a cube with a sign 0
   - The next one has a sign 1, and then one with sign 2
   - You conclude that cubes are associated to increasing natural numbers, where 0 is associated to the south-most cube
2. You are still walking on the bridge
   - You note that after cube 25 comes cube 27
   - You conclude that there is no cube 26
   - Perhaps, it may be an exception! (I really don't know)
3. Reached cube 31, someone asks you *where is cube 42?*
   - You have never seen cube 42
   - Still you can provide an answer
   - Cube 42 is north of here (or there is no cube 42!)

Understanding: Mode of reasoning (2)

- You push the on/off button, but no light turns on

- You buy a package of BeloCafe and obtain bad coffee
- Then, you buy a package of Guglielmo and obtain good coffee

Understanding: Implication (1)

$$L \to S$$

Sentences associated to propositions

L has libretto

S is student

If one has a libretto then they[1] is a student

L	S	$L \to S$
doesn't have libretto	is no student	OK!
has libretto	is no student	NO!
doesn't have libretto	is student	OK!
has libretto	is student	OK!

[1] Amazing British English stuff: Singular they!

Understanding: Implication (2)

Exercise

Which formula represents the following proposition?

Students are exactly those who have a libretto

L	S	$L \leftrightarrow S$
doesn't have libretto	is no student	OK!
has libretto	is no student	NO!
doesn't have libretto	is student	NO!
has libretto	is student	OK!

Understanding: Eliminate parentheses

Exercise 1.1 from *Logica a Informatica*

1. $((A \wedge B) \rightarrow (\neg C))$
2. $(A \rightarrow (B \rightarrow (\neg C)))$
3. $((A \wedge B) \vee (C \rightarrow C))$
4. $(\neg(A \vee ((\neg B) \rightarrow C)))$
5. $(A \rightarrow (B \vee (C \rightarrow D)))$
6. $(\neg((\neg(\neg(\neg A))) \wedge \bot))$
7. $(A \rightarrow (B \wedge ((\neg C) \vee D)))$

Where to place parentheses in the following one?

$$A \rightarrow B \rightarrow C$$

Tautologies and contradictions

Exercise 1.3 from *Logica a Informatica*

Decide whether the following formulas are tautologies or contradictions:

1. $(A \to (B \to C)) \to ((A \to B) \to (A \to C))$
2. $\neg(A \to \neg A)$
3. $A \vee \neg A$
4. $\bot \to A$
5. $\neg A \to (A \to B)$
6. $(A \wedge B) \wedge (\neg B \vee C)$
7. $A \vee B \to A \wedge B$
8. $(A \to C) \to ((B \to C) \to (A \vee B \to C))$
9. $(A \to B) \to ((B \to \neg C) \to \neg A)$

Which of these formulas are satisfiable?

Satisfiability

Similar to Exercise 1.4 from *Logica a Informatica*

Decide whether the following formula is satisfiable:

$$(A_1 \vee A_2) \wedge (\neg A_2 \vee \neg A_3) \wedge (A_3 \vee A_4) \wedge (\neg A_4 \vee A_5)$$

2-CNFs formulas, also known as Krom formulas,
can be solved in linear time!

Equivalences and consequences (1)

Exercise 1.8 from *Logica a Informatica*

Prove the following claims:

1. $\bot \vee B \equiv B$
2. $\neg\bot \wedge B \equiv B$
3. $A \models A$
4. $A \models B$ and $B \models C$ implies $A \models C$
5. $\models A \to B$ implies $A \wedge B \equiv A$ and $A \vee B \equiv B$
6. $\models A$ implies $A \wedge B \equiv B$
7. $\models A$ implies $\neg A \vee B \equiv B$
8. If $A \models B$ and $A \models \neg B$ then $\models \neg A$
9. If $A \models C$ and $B \models C$ then $A \vee B \models C$

What are A, B and C?

Equivalences and consequences (2)

Exercise 1.9 from *Logica a Informatica*

Check whether the following claims hold or not:

1. If $A \models B$ then $\neg A \models \neg B$
2. If $A \models B$ and $A \wedge B \models C$ then $A \models C$
3. If $A \vee B \models A \wedge B$ then $A \equiv B$

Normal Forms

Exercise 1.13 from *Logica a Informatica*

Find equivalent formulas in CNF for

1. $(A \to B) \to (B \to \neg C)$
2. $\neg(A \to (B \to \neg C)) \wedge D$
3. $\neg(A \wedge B \wedge (C \to D))$
4. $\neg(A \leftrightarrow B)$

Now find equivalent formulas in DNF!

Find the formula! (1)

Similar to Exercise 1.10 from *Logica a Informatica*

Find ϕ such that

A	B	ϕ
0	0	1
0	1	1
1	0	0
1	1	0

Using only $\rightarrow$ and $\bot$?

Find the formula! (2)

Similar to Exercise 1.13 from *Logica a Informatica*

Find ϕ such that

A	B	ϕ
0	0	1
0	1	0
1	0	0
1	1	0

Using only $\vee$ and $\neg$?

Find the formula in CNF and DNF!

Similar to Exercise 1.17 from *Logica a Informatica*

Find ϕs (one in CNF and one in DNF) such that

A	B	C	ϕ
0	0	0	1
0	0	1	0
0	1	0	1
0	1	1	0
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	0

Modelling dinner

Dinner constraints

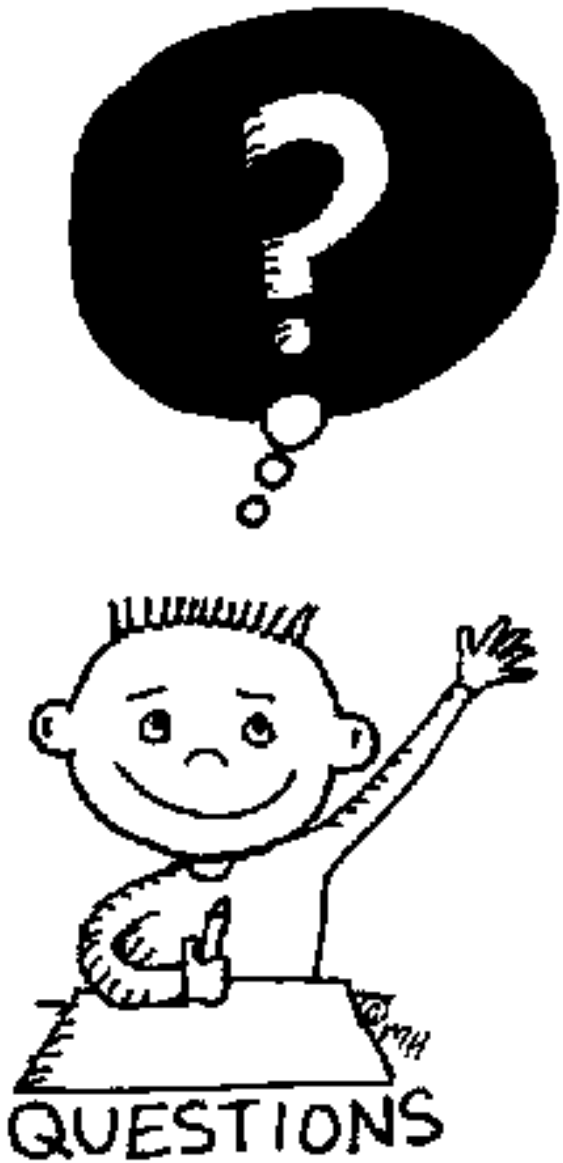
- Available dishes:
  1. Farfalle al salmone
  2. Risotto agli asparagi
  3. Tagliatelle ai funghi
  4. Filetto di manzo
  5. Spigola grigliata
  6. Trancia di pesce spada
- We can choose 7 white or 8 red wine
- We must choose exactly one primo, one secondo and one drink
- Do not eat fish after mushrooms
- Choose white wine if fish is involved

Goal: Write a set of wffs the models of which correspond to admissible dinner choices

Reduction to SAT

Reformulate the following questions such that they can be decided using a SAT algorithm:

1. Is $(P \vee (\neg P \rightarrow Q)) \leftrightarrow (P \vee Q)$ valid?
2. Does $P \rightarrow Q$ follow from $\neg Q \rightarrow \neg P$?
3. Is $P \leftrightarrow Q \wedge P$ a contradiction?
4. Is $P \leftrightarrow P \vee \bot$ a tautology?



END OF THE LECTURE