

Description Logics

Mario Alviano

University of Calabria, Italy

A.Y. 2018/2019

- 1 Introduction
- 2 Attributive concept Language with Complements
- 3 Extensions of ALC

Introduction

- Description Logics (DLs) are a family of knowledge representation languages
- The domain of discourse is described by **concept descriptions**
 - Atomic concepts and atomic roles are combined with the constructors provided by the particular DL
- DLs are equipped with a formal, logic-based semantics

Example

A human who is not female and who is married to a doctor, and all of whose children are either doctors or professors.

can be described as follows:

$$\text{Human} \sqcap \neg \text{Female} \sqcap (\exists \text{married}.\text{Doctor}) \sqcap (\forall \text{hasChild}.\text{Doctor} \sqcup \text{Professor})$$

- *ALC* stands for Attributive concept Language with Complements
- It is mainly focused on concepts
- It is the base for more expressive DLs
- *ALC* knowledge bases (KBs) consists of two parts
 - A terminological part, or TBox
 - An assertional part, or ABox

Let N_O be a set of object constants, or individuals (i.e., function symbols of arity 0).

Let N_C be a set of concept names (i.e., unary predicates).

Let N_R be a set of role names (i.e., binary predicates).

Concept description are defined inductively as follows:

- \top, \perp
- A , where $A \in N_C$
- $C \sqcap D, C \sqcup D, \neg C$, where C and D are concept description
- $\forall r.C, \exists r.C$, where C is a concept description and $r \in N_R$

An interpretation I is of the form (Δ^I, \cdot^I) , where

- Δ^I is a nonempty set called the domain of I
- \cdot^I maps every object constant o to an element o^I in Δ^I
- \cdot^I maps every concept name C to a subset C^I of Δ^I
- \cdot^I maps every role name r to a subset r^I of $\Delta^I \times \Delta^I$

An interpretation $I = (\Delta^I, \cdot^I)$ also gives a meaning to non-atomic concepts:

- $\top^I = \Delta^I, \perp^I = \emptyset$
- $(C \sqcap D)^I = C^I \cap D^I$
- $(C \sqcup D)^I = C^I \cup D^I$
- $(\neg C)^I = \Delta^I \setminus C^I$
- $(\exists r.C)^I = \{x \in \Delta^I \mid \text{there is } (x, y) \in r^I \text{ with } y \in C^I\}$
- $(\forall r.C)^I = \{x \in \Delta^I \mid \text{for all } (x, y) \in r^I \text{ it holds that } y \in C^I\}$

General concept inclusion (GCI)

Syntax:

A general concept inclusion (GCI) is of the form

$$C \sqsubseteq D$$

where C and D are concept

Semantics:

An interpretation I is a model of $C \sqsubseteq D$, denoted $I \models C \sqsubseteq D$, if $C^I \subseteq D^I$

Further notions

- $C \equiv D$ is an abbreviation for $C \sqsubseteq D$ and $D \sqsubseteq C$
- A **definition** is of the form $A \equiv C$, where $A \in N_C$

Syntax:

A TBox is a finite set of GCIs

Semantics:

An interpretation I is a model of a TBox \mathcal{T} , denoted $I \models \mathcal{T}$, if $I \models \phi$ holds for every GCI $\phi \in \mathcal{T}$

Further notions

A TBox is **definitorial** if

- it contains only definitions
- it contains at most one definition for any concept name
- it is acyclic, i.e., the definition of any $A \in N_C$ does not refer, directly or indirectly, to A itself

Syntax:

An assertional axiom is of one of the following forms:

$$a : C$$

$$(a, b) : r$$

where C is a concept, r is a role name, and a, b are object constants

Semantics:

Let I be an interpretation.

- $I \models a : C$ if $a^I \in C^I$
- $I \models (a, b) : r$ if $(a^I, b^I) \in r^I$

Syntax:

An ABox is a finite set of assertional axioms

Semantics:

An interpretation I is a model of an ABox \mathcal{A} , denoted $I \models \mathcal{A}$, if $I \models \phi$ holds for every assertional axiom $\phi \in \mathcal{A}$

Syntax:

A knowledge base (KB) K is a pair $(\mathcal{T}, \mathcal{A})$, where \mathcal{T} is a TBox and \mathcal{A} is an ABox

Semantics:

An interpretation I is a model of a KB $K = (\mathcal{T}, \mathcal{A})$, denoted $I \models K$, if $I \models \mathcal{T}$ and $I \models \mathcal{A}$

Let $K = (\mathcal{T}, \mathcal{A})$ be a KB.

- K is consistent if there is I such that $I \models K$
- A concept C is satisfiable wrt K if there is I such that $I \models K$ and $C^I \neq \emptyset$

As in first-order logic, we will write $K \models \phi$ if for every interpretation I such that $I \models K$ it holds that $I \models \phi$.

- The concept D subsumes the concept C wrt K if $K \models C \sqsubseteq D$
- C and D are equivalent wrt K if $K \models C \equiv D$
- An object a is an instance of a concept C wrt K if $K \models a : C$
- A pair of objects (a, b) is an instance of a role r wrt K if $K \models (a, b) : r$

Theorem

All inference problems in \mathcal{ALC} can be reduced to the consistency problem.

The same holds for any extension of \mathcal{ALC} .

Intuitively, all of these inference problems coincide with the entailment problem in first-order logic

Number restrictions

Syntax:

Number restrictions allows to describe concept of the forms

$$\leq k r$$

$$\geq k r$$

where k is a nonnegative integer and r is a role

Semantics:

- $(\leq k r)^I = \{x \in \Delta^I : |\{y \in \Delta^I : (x, y) \in r^I\}| \leq k\}$
- $(\geq k r)^I = \{x \in \Delta^I : |\{y \in \Delta^I : (x, y) \in r^I\}| \geq k\}$

Example

The following is a GCI asserting that a person can be married to at most one other individual:

$$Person \sqsubseteq \leq 1 \text{ married}$$

(Clearly, this GCI holds only for countries practicing monogamy)

Qualified number restrictions

Syntax:

Qualified number restrictions allows to describe concept of the form

$$\leq k r.C$$

where k is a nonnegative integer, r is a role, and C is a concept

Semantics:

- $(\leq k r.C)^I = \{x \in \Delta^I : |\{y \in C^I : (x, y) \in r^I\}| \leq k\}$
- $(\geq k r.C)^I = \{x \in \Delta^I : |\{y \in C^I : (x, y) \in r^I\}| \geq k\}$

- Inverse role

Syntax: r^- , where r is a role

Semantics: $(r^-)^I = \{(x, y) \in \Delta^I \times \Delta^I \mid (y, x) \in r^I\}$

- Transitive closure

Syntax: r^+ , where r is a role

Semantics: $(r^+)^I = r^I \cup \{(x, y) \in \Delta^I \times \Delta^I \mid \text{there is } z \in \Delta^I \text{ such that } (x, z) \in (r^+)^I \text{ and } (z, y) \in (r^+)^I\}$

- Reflexive transitive closure

Syntax: r^* , where r is a role

Semantics: $(r^*)^I = (r^+)^I \cup \{(x, x) \mid x \in \Delta^I\}$

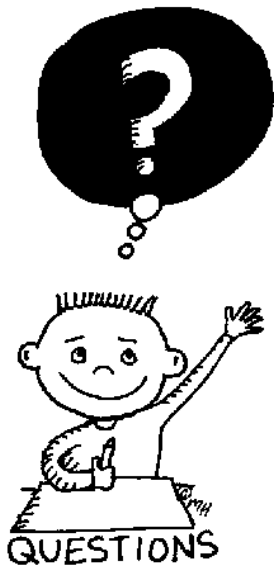
- Composition of roles

Syntax: $r \circ s$, where r, s are roles

Semantics: $(r \circ s)^I = \{(x, y) \in \Delta^I \times \Delta^I \mid \text{there is } z \in \Delta^I \text{ such that } (x, z) \in r^I \text{ and } (z, y) \in s^I\}$

- Subroles, i.e., axioms concerning roles (forming an RBox)
 - Syntax:** $r \sqsubseteq s$, where r, s are roles
 - Semantics:** $I \models r \sqsubseteq s$ if $r^I \subseteq s^I$
- As for GCIs, $r \equiv s$ is an abbreviation for $r \sqsubseteq s$ and $s \sqsubseteq r$

- `http://dl.kr.org/`
- `http://www.cs.man.ac.uk/~ezolin/dl/`



END OF THE
LECTURE