First-order logic
Tableau

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Outline

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First-order Tableau (1)

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Inference rules

$$\frac{\phi \wedge \psi}{\begin{array}{c}\phi \\ \psi\end{array}}\,(\wedge) \qquad \frac{\phi \vee \psi}{\phi \mid \psi}\,(\vee)$$

$$\frac{\forall x\ \phi}{\phi[t/x]}\,(\forall) \qquad \frac{\exists x\ \phi}{\phi[c/x]}\,(\exists)$$

Rule $(\forall)$ can be applied multiple times; t is any ground term.
In rule $(\exists)$, c is a fresh constant (Skolem term).

First-order Tableau (2)

Example: $\Gamma := \{\forall x\, P(x),\ \exists x(\neg P(x) \vee \neg P(f(x)))\}$

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First-order Tableau (2)

Example: $\Gamma := \{\forall x\, P(x),\ \exists x(\neg P(x) \lor \neg P(f(x)))\}$

$$\forall x\, P(x)$$
$$|$$
$$\exists x(\neg P(x) \lor \neg P(f(x)))$$
$$|$$
$$\neg P(c) \lor \neg P(f(c)))$$

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$$\neg P(c) \vee \neg P(f(c))$$
$$|$$
$$P(c)$$

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$$|$$

$$\neg P(c) \vee \neg P(f(c))$$

$$|$$

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$$\swarrow \qquad \searrow$$

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$$\neg P(c) \vee \neg P(f(c))$$

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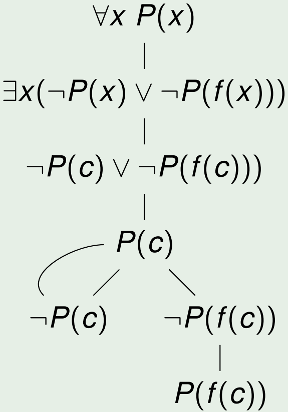

Rule ($\forall$) is applied multiple times!

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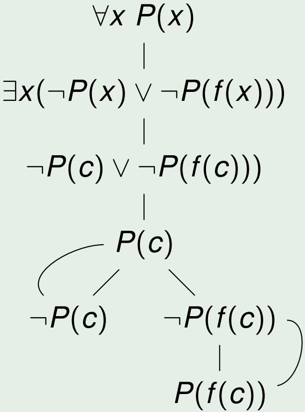

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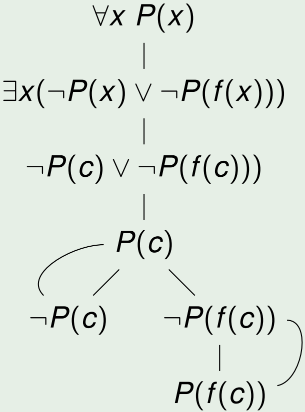

Rule (∀) is applied multiple times!

Γ is unsatisfiable!

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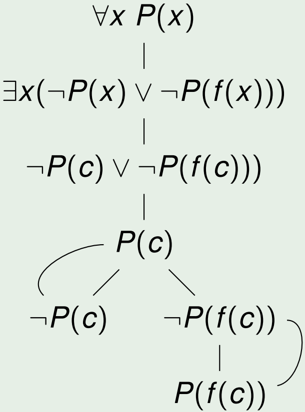


Rule ($\forall$) is applied multiple times!

Γ is unsatisfiable!

Theorem

First-order tableau is sound and complete.

First-order Tableau (3)

Examples

1. $\forall x\ P(x),\ \exists x\ \neg P(f(x))$
2. $\forall x\ \neg P(x),\ \exists x(P(x) \vee P(f(x)))$
3. $\forall x\ \neg P(x, a),\ P(a, b),\ \forall x \forall y(\neg P(x, y) \vee P(y, x))$

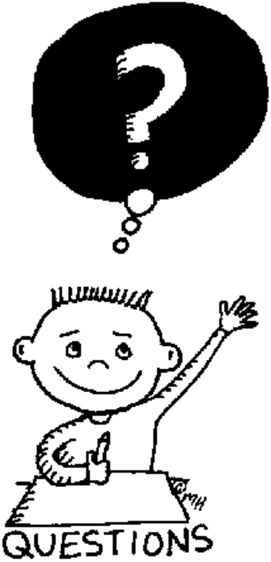
Outline

Exercises

1. Decide by means of the first-order tableau whether the following formulas (in SCNF) are satisfiable:
    1. $\{R(x, g(y))\}, \{\neg R(v, v), S(v)\}, \{\neg S(g(c))\}$
    2. $\{R(x, g(x))\}, \{\neg R(z, z), S(z)\}, \{\neg S(g(x))\}$
    3. $\{R(x, y), R(z, c), S(z)\}, \{\neg R(f(u), u), \neg R(v, w)\}, \{\neg S(f(d))\}$
2. Use the first-order tableau to prove that
    - $\forall x \forall y \forall z (P(x, y) \wedge P(y, z) \rightarrow P(x, z))$
    - $\forall x \forall y (P(x, y) \rightarrow P(y, x))$

   entail
    - $\forall x \forall y \forall z (P(x, y) \wedge P(z, y) \rightarrow P(x, z))$
3. Use the first-order tableau to prove that
    - $\forall x\, A(a, x, x)$
    - $\forall x \forall y \forall z (A(x, y, z) \rightarrow A(x, s(y), s(z)))$
    - $\forall x \forall y \forall z (A(x, y, z) \rightarrow A(y, x, z))$

   entail
    - $\exists x\, A(s(s(a)), s(s(s(a))), x)$



END OF THE LECTURE