Description Logics

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A.Y. 2018/2019

Outline

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  - Atomic concepts and atomic roles are combined with the constructors provided by the particular DL
- DLs are equipped with a formal, logic-based semantics

Example

A human who is not female and who is married to a doctor, and all of whose children are either doctors or professors.

can be described as follows:

$$Human \sqcap \neg Female \sqcap (\exists married.Doctor) \sqcap (\forall hasChild.(Doctor \sqcup Professor))$$

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- It is the base for more expressive DLs
- $\mathcal{ALC}$ knowledge bases (KBs) consists of two parts
  - A terminological part, or TBox
  - An assertional part, or ABox

$\mathcal{ALC}$ syntax

Let N_O be a set of object constants, or individuals (i.e., function symbols of arity 0).
Let N_C be a set of concept names (i.e., unary predicates).
Let N_R be a set of role names (i.e., binary predicates).

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Concept description are defined inductively as follows:

- $\top$, $\bot$
- A, where $A \in N_C$
- $C \sqcap D$, $C \sqcup D$, $\neg C$, where C and D are concept description
- $\forall r.C$, $\exists r.C$, where C is a concept description and $r \in N_R$

$\mathcal{ALC}$ semantics

An interpretation I is of the form $(\Delta^I, \cdot^I)$, where

- Δ^I is a nonempty set called the domain of I
- $\cdot^I$ maps every object constant o to an element o^I in Δ^I
- $\cdot^I$ maps every concept name C to a subset C^I of Δ^I
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An interpretation $I = (\Delta^I, \cdot^I)$ also gives a meaning to non-atomic concepts:

- $\top^I = \Delta^I$, $\bot^I = \emptyset$
- $(C \sqcap D)^I = C^I \cap D^I$
- $(C \sqcup D)^I = C^I \cup D^I$
- $(\neg C)^I = \Delta^I \setminus C^I$
- $(\exists r.C)^I = \{x \in \Delta^I \mid \text{there is } (x, y) \in r^I \text{ with } y \in C^I\}$
- $(\forall r.C)^I = \{x \in \Delta^I \mid \text{for all } (x, y) \in r^I \text{ it holds that } y \in C^I\}$

General concept inclusion (GCI)

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Further notions

- $C \equiv D$ is an abbreviation for $C \sqsubseteq D$ and $D \sqsubseteq C$
- A definition is of the form $A \equiv C$, where $A \in N_C$

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A TBox is a finite set of GCIs

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Further notions

A TBox is definitorial if

- it contains only definitions
- it contains at most one definition for any concept name
- it is acyclic, i.e., the definition of any $A \in N_C$ does not refer, directly or indirectly, to A itself

Assertional axiom

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An assertional axiom is of one of the following forms:

$$a : C$$

$$(a, b) : r$$

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Semantics:

Let I be an interpretation.

- $I \models a : C$ if $a^I \in C^I$
- $I \models (a, b) : r$ if $(a^I, b^I) \in r^I$

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A knowledge base (KB) K is a pair $(\mathcal{T}, \mathcal{A})$, where $\mathcal{T}$ is a TBox and $\mathcal{A}$ is an ABox

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Inference problems (1)

Let $K = (\mathcal{T}, \mathcal{A})$ be a KB.

- K is consistent if there is I such that $I \models K$
- A concept C is satisfiable wrt K if there is I such that $I \models K$ and $C^I \neq \emptyset$

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- C and D are equivalent wrt K if $K \models C \equiv D$
- An object a is an instance of a concept C wrt K if $K \models a : C$
- A pair of objects (a, b) is an instance of a role r wrt K if $K \models (a, b) : r$

Inference problems (2)

Theorem

All inference problems in $\mathcal{ALC}$ can be reduced to the consistency problem.
The same holds for any extension of $\mathcal{ALC}$.

Intuitively, all of these inference problems coincide with the entailment problem in first-order logic

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Number restrictions allows to describe concept of the forms

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Semantics:

- $(\leq k\ r)^I = \{x \in \Delta^I : |\{y \in \Delta^I : (x, y) \in r^I\}| \leq k\}$
- $(\geq k\ r)^I = \{x \in \Delta^I : |\{y \in \Delta^I : (x, y) \in r^I\}| \geq k\}$

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Example

The following is a GCI asserting that a person can be married to at most one other individual:

$$Person \sqsubseteq \leq 1\ married$$

(Clearly, this GCI holds only for countries practicing monogamy)

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Defined roles

- Inverse role
  Syntax: r^-, where r is a role
  Semantics: $(r^-)^I = \{(x, y) \in \Delta^I \times \Delta^I \mid (y, x) \in r^I\}$

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  Syntax: r^-, where r is a role
  Semantics: $(r^-)^I = \{(x, y) \in \Delta^I \times \Delta^I \mid (y, x) \in r^I\}$
- Transitive closure
  Syntax: r^+, where r is a role
  Semantics: $(r^+)^I = r^I \cup \{(x, y) \in \Delta^I \times \Delta^I \mid \text{there is } z \in \Delta^I$ such that $(x, z) \in (r^+)^I$ and $(z, y) \in (r^+)^I\}$

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- Reflexive transitive closure
  Syntax: r^*, where r is a role
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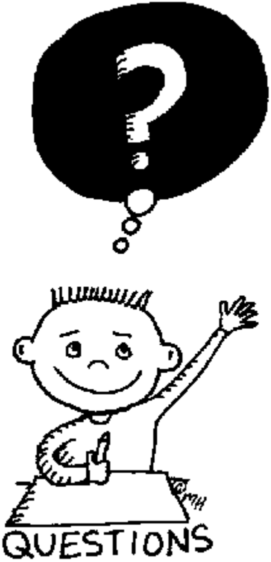
- Reflexive transitive closure
  Syntax: r^*, where r is a role
  Semantics: $(r^*)^I = (r^+)^I \cup \{(x, x) \mid x \in \Delta^I\}$
- Composition of roles
  Syntax: $r \circ s$, where r, s are roles
  Semantics: $(r \circ s)^I = \{(x, y) \in \Delta^I \times \Delta^I \mid$ there is $z \in \Delta^I$ such that $(x, z) \in r^I$ and $(z, y) \in s^I\}$

Role axioms

- Subroles, i.e., axioms concerning roles (forming an RBox)
  Syntax: $r \sqsubseteq s$, where r, s are roles
  Semantics: $I \models r \sqsubseteq s$ if $r^I \subseteq s^I$
- As for GCIs, $r \equiv s$ is an abbreviation for $r \sqsubseteq s$ and $s \sqsubseteq r$

Useful resources on the web

- http://dl.kr.org/
- http://www.cs.man.ac.uk/~ezolin/dl/



END OF THE LECTURE