Process Mining in Complex Domains

joint work with Antonella Guzzo and Luigi Pontieri
An Application Domain
An Application Domain
Gioia Tauro

- Italian harbor acting as a maritime freight hub (about 4 millions of containers per year).
  - Berth planning
  - Routing
  - …
  - Yard planning
The mission is to offer high quality of service to the navigation lines, while reducing the overall cost of internal logistic processes.

Critical performance measures are

- *the latency time* elapsed when serving a ship (where, typically, a number of containers are both discharged off and charged on), and
- *the overall costs* of moving the containers around the yard.

A key factor impacting on both these measures is the number of “*house-keeping*” moves that are applied to the containers.
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Minimize house-keeping moves
Yard Planning

- The mission is to offer high quality of service to the navigation lines, while reducing the overall cost of internal logistic processes.

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Minimize house-keeping moves

Understand the process, first!
The yard

slot
The yard

block
The yard

sector
The yard
The container is initially unloaded from the ship, with the help of a crane.

It is first stocked within a zone near to the dock.

It is carried to some slot of the yard, via:
- cranes
- straddle-carriers (a vehicle capable of picking and carrying a container, by possibly lifting it up)
- multi-trailers (a sort of train-like vehicle that can transport many containers)

At boarding time, the container is first placed in a yard area close to the dock.

Finally, it is loaded on the cargo by means of a crane.
Challenges

- Logs from transactional systems
- Logs mix different usage scenarios
- Traces are stored at different level of details
- Noise
- Huge volume of data
Outline

Application Domain

Process Mining Approaches @UniCAL

Another Challenge in Process Mining

Formal Framework

Implementation Issues
(1) Clustering

- Log traces
- CF Graph induction
- Basic WF Schema
- Partitioning & Refinement
- Disjunctive Workflow Schema
(1) Clustering

PF Graph induction

log traces

basic WF Schema

Partitioning & Refinement

Disjunctive Workflow Schema

Discriminant rules:

<table>
<thead>
<tr>
<th>r_1</th>
<th>r_2</th>
<th>r_n</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.3</td>
<td>0.4</td>
</tr>
<tr>
<td>0.1</td>
<td>0.1</td>
<td>0.1</td>
</tr>
</tbody>
</table>

Projection
(2) Abstraction

The tree describes the process behavior at different level of details.

At the highest level of detail (leaves of the tree), the schemas could be used to support the design of concrete workflow models.

At lower levels, the schemas are abstract views over heterogeneous behaviors, which could support analysis and monitoring tasks.

**Basic Idea:**

1. The hierarchy is restructured bottom-up at different levels.
2. Produce an abstraction dictionary.
Basic idea:

- find a comprehensive representation for the process, describing both structural and non-structural aspects
- A rule-based classifier is induced to discriminate among given structural clusters, based on process/task data
- help interpreting/predicting the different ways of executing the process, based on properties of process/task instances
(4) Outlier Detection

- Structural patterns are identified

- **fork s-pattern**

- **join s-pattern**
(4) Outlier Detection

- Structural patterns are identified.
- They are co-clustered with the traces, based on what extent these latter support them.
- Mark as outlier each trace \( t \) such that either:
  - \( t \) has not been assigned to any cluster.
  - \( t \) belongs to a cluster whose cardinality is "appreciably smaller" than the average cluster size.
Process Mining

Requirements → Process Design → Implementation
Process Mining

Requirements → Process Design → Implementation

Process Mining

abcdfg
abcdef
...
Process Mining

Requirements → Process Design → Implementation

Process Mining

abcdfg
abcfdf
abcdfe

Diagram showing the process flow from requirements to implementation with Process Mining at different stages.
Process Mining

Requirements → Process Design → Implementation

Process Mining

abcdfg
abcfd
abcdfe
...

Diagram showing the process mining flow from requirements to implementation.
Process Mining + Background Knowledge

Requirements → Process Design → Implementation

Process Mining

abcdfg
abcf
abcdfe

...
Process Mining + Background Knowledge

- Requirements
- Process Design
- Implementation

Process Mining

Knowledge + Big Data

More Knowledge
**Dependency Graph:** directed graphs whose nodes one-to-one correspond with the activities and such that an edge from an activity $a$ to an activity $b$ means that, in some enactment we expect that an actual flow of information can occur from $a$ to $b$. 
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**Example**

\( abcde \)

\( acbde \)
**Dependency Graph:** directed graphs whose nodes one-to-one correspond with the activities and such that an edge from an activity $a$ to an activity $b$ means that, in some enactment we expect that an actual flow of information can occur from $a$ to $b$.

**Example**

<table>
<thead>
<tr>
<th>$G_1$</th>
<th>{abcde,acbde}</th>
</tr>
</thead>
<tbody>
<tr>
<td>$G_2$</td>
<td>{abcde,acbde}</td>
</tr>
</tbody>
</table>
**Process Models**

*Dependency Graph:* directed graphs whose nodes one-to-one correspond with the activities and such that an edge from an activity $a$ to an activity $b$ means that, in some enactment we expect that an actual flow of information can occur from $a$ to $b$.

**Example**

- $G_0 \times \{abcde, acbde\}$
- $G_1 \vdash \{abcde, acbde\}$
- $G_2 \vdash \{abcde, acbde\}$
Two kinds of positive constraints $\pi$:

- edge constraint $S \rightarrow a$
- path constraint $S \rightsquigarrow a$

where $S \subseteq \mathcal{A}$, with $|S| \geq 1$, is a non-empty set of activities and $a \in \mathcal{A} \setminus S$ is an activity.

For a positive constraint $\pi$, $\neg\pi$ is a negative precedence constraint.
Precedence Constraints

Syntax

- *edge constraint* $S \rightarrow a$
- *path constraint* $S \leadsto a$

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Semantics is interpreted over *directed graphs*.
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\begin{align*}
\{a,c\} & \rightarrow d \\
\{b\} & \rightarrow c
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**Semantics** is interpreted over *directed graphs*

- $\{a,c\} \rightarrow d$
- $\{b\} \rightarrow c$
- $\{c,b\} \rightsquigarrow e$
**Precedence Constraints**

**Syntax**
- *edge constraint* \( S \rightarrow a \)
- *path constraint* \( S \leadsto a \)

*For a positive constraint \( \pi \), \( \neg \pi \) is a negative precedence constraint.*

**Semantics** is interpreted over *directed graphs*

- \( \{a,c\} \rightarrow d \)
- \( \{b\} \rightarrow c \) (red underline)
- \( \{c,b\} \leadsto e \)
- \( \{c\} \leadsto b \)
Precedence Constraints

Syntax

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Semantics is interpreted over *directed graphs*

```
{a,c} → d
{b} → c
{c,b} ≺ e
{c} ―→ b
```
DG-MINING: Given a log $L$ and a set $\Pi$ of precedence constraints over $A(L)$, compute a dependency graph $G$ for $L$ with $G \models \Pi$. 
Revisiting Process Discovery

**DG-MINING:** Given a log $L$ and a set $\Pi$ of precedence constraints over $A(L)$, compute a dependency graph $G$ for $L$ with $G \models \Pi$.

**ACYCLIC-DG-MINING:** Given a log $L$ and a set $\Pi$ of precedence constraints over $A(L)$, compute an acyclic dependency graph $G$ for $L$ with $G \models \Pi$. 
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**Acyclic-DG-Mining:** Given a log $L$ and a set $\Pi$ of precedence constraints over $A(L)$, compute an acyclic dependency graph $G$ for $L$ with $G \models \Pi$.

\[
\begin{align*}
\Pi_0 &= \{ \neg(b \leadsto d), \neg(d \leadsto b) \} \\
t_0 &= abcde
\end{align*}
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$$
\Pi_0 = \{ \neg\{(b) \rightsquigarrow d\}, \neg\{(d) \rightsquigarrow b\}\}
$$

$$
t_0 = abcde
$$

$G_2$ is a solution to DG-MINING (on input $\{t_0\}$ and $\Pi_0$)
As a result of our formulation, process discovery is conceptually carried out via:

- a *learning task* (i.e., building all possible dependency graphs for a given input log), followed by
- a *reasoning task* (i.e., to filter out those graphs that do not satisfy the precedence constraints defined by the analyst)
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Exponentially many dependency graphs might be built in the learning phase.
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- a *learning task* (i.e., building all possible dependency graphs for a given input log), followed by
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A two-phase approach is unfeasible.
Let $L$ be a log. For each trace $t[1]...t[n] \in L$,

$$
\pi(t) = \{ \{t[1],...,t[i-1]\} \rightarrow t[i] \mid 1 < i \leq n \}
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Moreover, let $\pi(L) = \bigcup_{t \in L} \pi(t)$. 
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**Theorem**

$G$ is a solution to DG-Mining (resp., ACYCLIC-DG-Mining) on input $L$ and $\Pi$

$$G \models \pi(L) \cup \Pi.$$
Complexity Analysis

DG-Mining

{→, ∼, ⊥}
{→, ⊥, ∼}
{→, ∼}
{⊥, ⊥}
{→, ⊥}
{∼, ⊥}
{⊥}
{∼}

Acyclic-DG-Mining

NP-hard
Polynomial Time
Complexity Analysis
Outline

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Formally, a **CSP instance** is a triple \((Var, U, C)\), where

- \(Var = \{X_1,\ldots,X_m\}\) is a finite set of variables
- \(U\) is a function mapping each variable \(X_i \in Var\) to a domain \(U(X_i)\) of values
- \(C\) is a finite set of constraints, i.e., boolean functions over \(\{X_{i1},\ldots,X_{ik}\}\), such as:
  - summation constraints, of the form: \(\Sigma w_j \times X_j \geq \gamma\)
  - reified (summation) constraints, of the form: \(\Sigma w_j \times X_j \geq \gamma \iff X\)

... where \(X\) is a boolean variable, while \(\gamma\) and all \(w_j\) are real numbers.
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An **assignment** \(\theta\) for the CSP instance \((\text{Var}, U, C)\) is a function mapping each variable \(X_i \in \text{Var}\) to an element of its associated domain \(U(X_i)\).

- \(\theta\) is a **solution** to \((\text{Var}, U, C)\) if it satisfies all the constraints in \(C\).
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- \(\theta\) is a solution to \((V, U, C)\) if it satisfies all the constraints in \(C\).

In addition to constraints, in a CSOP instance, an optimal solution is searched, minimizing a linear cost function of the form

\[ f(\theta) = \sum_{i=1}^{n} w_i \times \theta(X_i) \]
Basic encoding algorithm \textit{PCtoCSP}

A given set of precedence constraints over activities \{a_1, ..., a_n\} is encoded into a CSP instance, containing a series of variables for each pair \(a_i\) and \(a_j\) of activities:

- an “edge” variables \(e[a_i,a_j]\),
- “path” variables \(p[a_i,a_j]^l\) and “path-through” variables \(p[a_i,a_k,a_j]^l\), for \(k,l = 1..n\), where \(l\) denotes the maximum number of edges in the respective path.
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As a basic solution scheme, PC-CSP uses backtracking, while PC-CSOP uses a branch-and-bound approach.

During the exploration, all solution algorithms alternate two kinds of steps:
- \textit{branching}, where a value is assigned to some variables as in standard search methods, and
- \textit{constraint propagation}, where different constraints can be iteratively applied as to shrink the space of the possible dependency graphs and propagate the consequences of choices made in the previous steps.
**CP solving algorithms**

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Ad-hoc branching policies have been implemented:

- we first branch on the edge variables before considering path variables;
- all variables $p[a_i,a_j]$ are always considered before those of the form $p[a_i,a_k,a_j]$.
Heuristics

In order to pragmatically reduce the size of the search space and speed-up the computation, three types of heuristics can be used:

1. **Redundancy Reduction.** Two policies, relying on two different notions of constraint subsumption:
   - A constraint $S \rightarrow a$ is filtered out if there is another precedence constraint $S' \rightarrow a$ such that $S' \supset S$
   - A constraint $S \rightarrow a$ is filtered out if there exist another constraint $S'' \rightarrow a$ such that $S'' \subset S$
     - this (weaker) notion allows for recognizing skip-like control flow structures, where some synchronizing (i.e. join) activity $a$ can be activated by an activity in $S \setminus S''$ or, optionally, by an activity in $S \cap S''$. 
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2. **Closed World Assumption (CWA).** In order to reduce the size of the search space, further constraints are introduced as follows:
   - an edge $(x, y)$ is not permitted to appear in the model if activity $y$ never follows activity $x$, (directly or indirectly), in any trace of the log.
   - In the case of unfolding, CWA constraints are expressed over real activities, rather than on their unfolded versions.
The **nr. of elements in constraint bodies** is a key factor for scalability.

Let \( \{ t[1], ..., t[i-1] \} \rightarrow t[i] \) be a constraint in the set \( \pi(t) \) of precedence constraints derived from a given trace \( t[1], ..., t[n] \).

Three strategies for shrinking the size of the body (i.e., left hand part):

a) **Maximal horizon \( H \) over past activities:**
   - remove each \( t[j] \) s.t. \( j < i-H \), from \( \{ t[1], ..., t[i-1] \} \)

b) **Two kinds of lower thresholds** for edge weights: \( \sigma_{abs} \) (“absolute”), and \( \sigma_{r2b} \) (“relative to best predecessor”, like in Heuristics Miner)
   - remove any \( t[j] \) such that \( weight(t[j],t[i]) < \sigma_{abs} \)
   - remove any \( t[j] \) s.t. \( weight(t[j],t[i]) < \sigma_{r2b} \times \text{argmax}_{1 \leq k < l} \{ weight(t[k], t[i]) \} \)

c) **Maximum number \( K_{top} \) of activities that can occur in the body:**
   - at most \( K_{top} \) elements are kept, with top dependency scores (w.r.t. \( t[j] \))
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Thank you!