Logical Implications of Non Locality Proofs in QM

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AIM OF THE WORK:

Real implications of non-locality theorems

On

The Consistency between QM and Locality

Locality. $\mathcal{R}_\alpha$ spacelike separated from $\mathcal{R}_\beta$

implies that

Reality in $\mathcal{R}_\beta$ unaffected by operations in $\mathcal{R}_\alpha$

Theorems: QM inconsistent with locality:

Archetypal: Bell [1964], Greenberger, Horne, Zeilinger [1989]

Hardy [1993]

Proofs require QM and furhter conditions

(i) reality of unmeasured quantities

(ii) extensions of QM correlations

$\uparrow$ motivation

Criterion of Reality: *If, without in any way disturbing a system, we can predict with certainty the value of a physical quantity, then there exists an element of physical reality corresponding to this physical quantity.*

Analysis of archetypal proofs:

The extensions of correlations,
actually deducible from the criterion of reality
are not large enough to ensure
validity of the three archetypal proofs.

Foundations of Physics, **41** (2011), 1263

Theorems different from archetypal ones

Proofs of inconsistency without (i) and (ii)

Stapp, Am.J.Phys., **72** (2004) 30

no values of unmeasured observables

no Criterion of Reality, no Hidden Variables

but instead,

(NBITI) No Backward in Time Influence

(FC) Free Choice (*free will?*)

General Formalism

Hilbert space $\mathcal{H}$, $|\psi\rangle \in \mathcal{H}$ state vector.

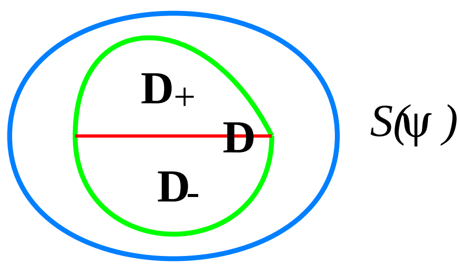
Support: $\mathcal{S}(|\psi\rangle)$ Concrete set of specimens whose state is $|\psi\rangle$

Observable $\mathcal{D} \longrightarrow \widehat{D}$ its self-adjoint operator

$\mathcal{D}$ is *two-value* if $\sigma(\widehat{A}) = \{-1, +1\}$.

$\mathbf{D} = \{x \in \mathcal{S}(|\psi\rangle) \mid \mathcal{D}$ is measured on $x\}$

$\mathbf{D}_{\pm} = \{x \in A \mid$ outcome$= \pm 1\}$



Properties

(q.i) $\forall|\psi\rangle\ \exists\mathcal{S}(|\psi\rangle)$ such that $\mathbf{D} \neq \emptyset$

(q.ii) $\mathbf{D}_{+} \cap \mathbf{D}_{-} = \emptyset$, (q.iii) $\mathbf{D}_{+} \cup \mathbf{D}_{-} = \mathbf{D}$

(q.iv) $[\hat{D}, \hat{B}] = \mathbf{0} \Rightarrow \exists\mathcal{S}(|\psi\rangle)$ s.t. $\mathbf{D} \cap \mathbf{B} \neq \emptyset$

(q.v) $[\hat{D}, \hat{B}] \neq \mathbf{0} \Rightarrow \mathbf{D} \cap \mathbf{B} = \emptyset, \quad \forall\mathcal{S}(|\psi\rangle)$

Empirical Implication: $\mathcal{D} \rightarrow \mathcal{B}$

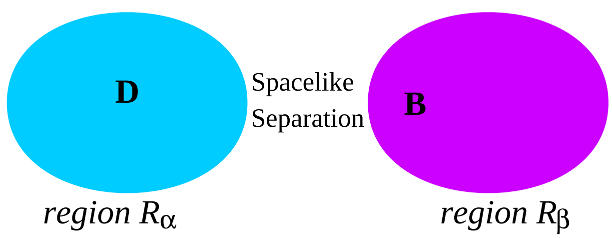
(outcome of $\mathcal{D}$)= 1 $\Rightarrow$ (outcome of $\mathcal{B}$)=1

(E.1) $\quad [\widehat{D}, \widehat{B}] = \mathbf{0}$,

(E.2) $\forall x \in \mathbf{D} \cap \mathbf{B}$, $x \in \mathbf{D}_{+} \Rightarrow x \in \mathbf{B}_{+}$

$$\left((E.2) \Leftrightarrow \quad \frac{1+\widehat{D}}{2}\frac{1+\widehat{B}}{2}|\psi\rangle = \frac{1+\widehat{D}}{2}|\psi\rangle \right)$$

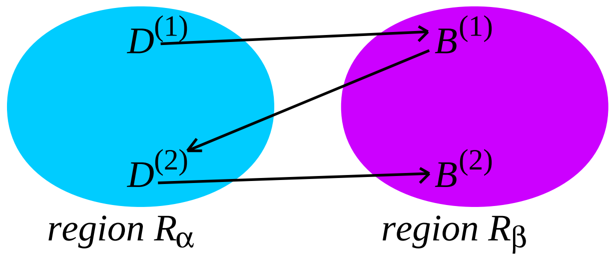
Separated Observables: $\mathcal{D} \bowtie \mathcal{B}$



Stapp's Theorem

Hypotheses:

QM predictions for Hardy's experimental setting



No Criterion of realitiy (conditions (i), (ii))

New further hypotheses:

(NBITI): no backward in time influence

(FC): Free Choice

The argument

Statement (SR): A law within $\mathcal{R}_\beta$

PROP.1. If $\mathcal{D}^{(2)}$ is measured in $\mathcal{R}_\alpha$ then **(SR)** holds in $\mathcal{R}_\beta$.

PROP.2. If $\mathcal{D}^{(1)}$ is measured in $\mathcal{R}_\alpha$ then **(SR)** does not hold in $\mathcal{R}_\beta$.

VIOLATION OF LOCALITY CONDITION!

A CLOSER LOOK

Further Conditions

(FC) "This premise asserts that the choice made in each region as to which experiment will be performed in that region can be treated as a localized free variable."

(NBITI) "This premise asserts that experimental outcomes that have already occurred in an earlier region [...] can be considered fixed and settled independently of which experiment will be chosen and performed later in a region spacelike separated from the first."

(SR) Statement (*Physical law*) within $\mathcal{R}_\beta$:

If $\mathcal{B}^{(1)}$ *is measured and the outcome* $+1$ *is obtained, then if instead* $\mathcal{B}^{(2)}$ *had been measured the outcome would have been* $+1$*.*

Prop.1. $\mathcal{D}^{(2)}$ measured in $\mathcal{R}_\alpha \Rightarrow$ **(SR)** in $\mathcal{R}_\beta$.

Proof of Prop.1: “The concept of ‘instead’ [in (SR)] is given a unambiguous meaning by the combination of the premises of ‘free’ choice and ‘no backward in time influence’; the choice between [$\mathcal{B}^{(2)}$] and [$\mathcal{B}^{(1)}$] is to be treated, within the theory, as a free variable, and switching between [$\mathcal{B}^{(2)}$] and [$\mathcal{B}^{(1)}$] is required to leave any outcome in the earlier region [$\mathcal{R}_\alpha$] undisturbed. But the statements [Hardy’s correlations] can be joined in tandem to give the result (SR)”

PRESENT WORK

1. Formulate **(NBITI)** and **(FC)** as formal statements within a coherent theory, so that

2. the validity of **Prop.1** and **Prop. 2** can be verified according to universal logico-mathematical criteria.

1. (NBITI) and (FC) as formal statements

Example: $\widehat{B}$ represents $\mathcal{B}$ at t_2.

If $x \in \mathcal{S}(|\psi\rangle)$ and $\widehat{B}|\psi\rangle = |\psi\rangle$

then the following prediction made at $t < t_2$

a measurement at t_2 of $\mathcal{B}$ yields outcome +1

is valid by (FC): $x \in \mathbb{B}_+$

$x \in \mathbb{B}_+$ ascribes no physical reality to $\mathcal{B}$!

General properties

(C.1) $\mathbb{B}_+ \cap \mathbb{B}_- = \emptyset$,

(C.2) $x \in \mathbb{B}_- \Rightarrow x \notin \mathbf{B}_+$ and $x \in \mathbb{B}_+ \Rightarrow x \notin \mathbf{B}_-$

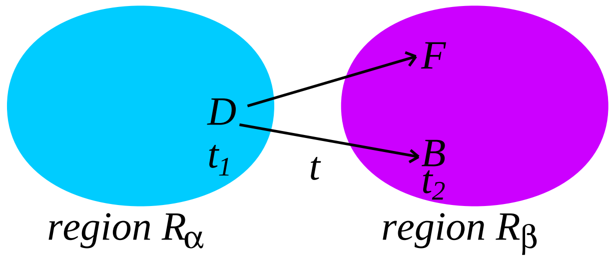
More

Let $\mathcal{D} \bowtie \mathcal{F}$ and $D \longrightarrow F$. Suppose that $x \in \mathbf{D}_+$.

If $x \in \mathbf{F}$ (time t_2), the prediction made at t

$$\{\textit{the outcome is } +1\} \equiv (x \in \mathbb{F}_+)$$

is valid in QM: $x \in \mathbf{D}_+ \cap \mathbf{F}$ and $\mathcal{D} \rightarrow \mathcal{F}$, (NBITI).



Let $\mathcal{B}$ be so that $\mathcal{D} \rightarrow \mathcal{B}$.

By (FC) $\mathcal{B}$ and $\mathcal{F}$ are *equivalent* at t: $x \in \mathbb{B}_+$.

(C.3) If $\mathcal{D} \bowtie \mathcal{B}$, $\mathcal{D} \rightarrow \mathcal{B}$ then $x \in \mathbf{D}_+ \Rightarrow x \in \mathbb{B}_+$.

Remark

Stronger Statement:

If $\mathcal{D} \bowtie \mathcal{B}$, $\mathcal{D} \to \mathcal{B}$ then $x \in \mathbb{D}_{+} \Rightarrow x \in \mathbb{B}_{+}$

NOT DEDUCIBLE FROM (NBITI) and (FC):

If $x \in \mathbf{D}_{+}$ does not hold

No actual outcome of $\mathcal{D}$ is available

for invoking QM prediction

of the otcomes of $\mathcal{F}$ or $\mathcal{B}$!

(NBITI) and (FC) supplement QM with

$$x \in \mathbb{B}_{\pm}$$

validity of the prediction at $t < t_2$

the outcome of a $\mathcal{B}$ measurement at t_2 is $+1$.

Coherently with (NBITI), (FC), QM

(C.1) $\mathbb{B}_{+} \cap \mathbb{B}_{-} = \emptyset$,

(C.2) $x \in \mathbb{B}_{-} \Rightarrow x \notin \mathbb{B}_{+}$ and $x \in \mathbb{B}_{+} \Rightarrow x \notin \mathbb{B}_{-}$

(C.3) If $\mathcal{D} \bowtie \mathcal{B}$, $\mathcal{D} \to \mathcal{B}$ then $x \in \mathbf{D}_{+} \Rightarrow x \in \mathbb{B}_{+}$.

Reformulation

Statement (SR):

If $\mathcal{B}^{(1)}$ is measured and the outcome $+1$ is obtained, then if instead $\mathcal{B}^{(2)}$ had been measured the outcome would have been $+1$.

translates into

$x \in \mathbf{B}_+^{(1)}$ implies $x \in \mathbb{B}_+^{(2)}$

THEOREM

Hyptheses: QM predictions, (C.1),(C.2),(C.3)

Prop.1. $x \in \mathbf{D}^{(2)}$ implies **(SR)** in $\mathcal{R}_\beta$

Prop.2. $x \in \mathbf{D}^{(1)}$ implies $\neg$**(SR)** in $\mathcal{R}_\beta$

SECOND TASK:
ANALYSIS OF THE PROOF

HYPOTHESES: QM predictions

h.i) $\{\mathcal{D}^{(1))}, \mathcal{D}^{(2))}\} \bowtie \{\mathcal{B}^{(1)}, \mathcal{B}^{(2)}\}$,

$|\psi\rangle$ not eigenvector.

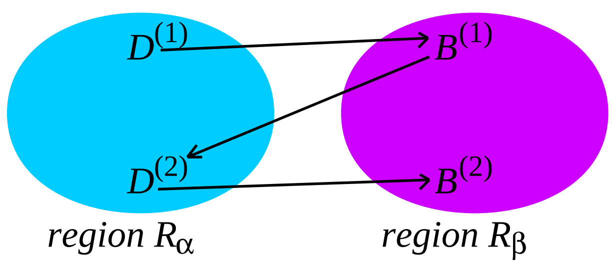
h.ii) $[\widehat{D}^{(1)}, \widehat{D}^{(2)}] \neq \mathbf{0}$, $[\widehat{B}^{(1)}, \widehat{B}^{(2)}] \neq \mathbf{0}$

h.iii) $\mathcal{D}^{(1)} \rightarrow \mathcal{B}^{(1)} \rightarrow \mathcal{D}^{(2)} \rightarrow \mathcal{B}^{(2)}$,

$([\widehat{D}^{(j)}, \widehat{B}^{(k)}] = \mathbf{0})$.

h.iv) $\exists \mathcal{S}(|\psi\rangle)$ and $x_0 \in \mathcal{S}(|\psi\rangle)$, $x_0 \in \mathbf{D}_{+}^{(1)} \cap \mathbf{B}_{-}^{(2)}$

([*probability*$(D^{(1)} = 1 \& B^{(2)} = -1) \neq 0$)



(NSR) $x \in \mathbf{B}_+^{(1)}$ implies $x \in \mathbb{B}_+^{(2)}$

Prop.1. $x \in \mathbf{D}^{(2)}$ implies **(NSR)** in $\mathcal{R}_\beta$

$$\forall x \in \mathbf{D}^{(2)},\ x \in \mathbf{B}_+^{(1)} \Rightarrow x \in \mathbb{B}_+^{(2)}$$

Prop.2. $x \in \mathbf{D}^{(1)}$ implies $\neg$**(NSR)** in $\mathcal{R}_\beta$

$\exists \mathcal{S}(|\psi\rangle), x_0 \in \mathbf{D}^{(1)},\ x_0 \in \mathbf{B}_+^{(1)}$ but $x_0 \notin \mathbb{B}_+^{(2)}$

Reformulation of the proof of Prop.1

(P.1.i) suppose that $x \in \mathbf{D}^{(2)} \cap \mathbf{B}_+^{(1)}$ (S.1)

(P.1.ii) (S.1) implies $x \in \mathbf{B}^{(1)} \cap \mathbf{D}^{(2)}$ (S.2)

(P.1.iii) Then (h.iii), (S.1), (S.2), (E.1), imply $x \in \mathbf{D}_+^{(2)}$ (S.3)

(P.1.iv) (h.iii), (S.3) and (C.3) imply $x \in \mathbb{B}_+^{(2)}$.

Its validity can be verified:

THIS PROOF IS VALID
[http:arxiv.org/pdf/1111.5121.pdf]

Reformulation of the proof of Prop.2

(P.2.i) $\mathbf{D}_{+}^{(1)} \neq \emptyset$, for some $\mathcal{S}(|\psi\rangle)$

(P.2.ii) $x \in \mathbf{D}_{+}^{(1)} \Rightarrow x \in \mathbf{B}_{+}^{(1)}$

(P.2.iii) Antecedent of (NSR) holds $\forall x \in \mathbf{D}_{+}^{(1)}$.

(P.2.iv) $\exists x_0 \in \mathbf{D}_{+}^{(1)}$ such that $x_0 \in \mathbf{B}_{-}^{(2)}$

(P.2.v) $x_0 \notin \mathbb{B}_{+}^{(2)}$:

$$\exists x_0 \in \mathbf{D}^{(1)},\ x_0 \in \mathbf{B}_{+}^{(1)} \text{ and } x \notin \mathbb{B}_{+}^{(2)}$$

The analysis of this proof shows that

IT IS NOT VALID!
[http:arxiv.org/pdf/1111.5121.pdf]

CONCLUSION

Quantum Mechanics has been coherently supplemented with formal statements implied by the assumptions (NBITI) and (FC).

Within the supplemented theory
Prop.1 is valid

Prop.2 is not valid.

Thus, according to this approach,

Consistency between Quantum Mechanics and Locality is not affected by Stapp's argument