Univerität Basel

Herbsemester 2012

Master course A. Surroca - L. Paladino

Some topics on modular functions, elliptic functions and transcendence theory

Sheet of exercises n.2

2.1. Show that

$$\mathrm{Im}(\gamma \cdot \tau) = \frac{\mathrm{Im}(\tau)}{\mid c\tau + d \mid^2}, \quad \text{for all } \gamma = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in \mathrm{SL}_2(\mathbb{Z}).$$

2.2. Show that

$$(\gamma\gamma') \cdot \tau = \gamma \cdot (\gamma' \cdot \tau), \quad \text{for all } \gamma, \gamma' \in \mathrm{SL}_2(\mathbb{Z}) \text{ and } \tau \in \mathcal{H}.$$

2.3. Let

$$S = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}, \qquad T = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$$

and let $\Gamma' =< S, T >$ be the subgroup of $\Gamma = \mathrm{SL}_2(\mathbb{Z})$ spanned by S and T. For every

$$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in \Gamma,$$

a) calculate $A' = AT^n = \begin{pmatrix} a' & b' \\ c' & d' \end{pmatrix}$, for all $n \in \mathbb{Z}$;

b) show that there exists $n \in \mathbb{Z}$ such that $|d'| \leq |c|/2$;

c) calculate $A'' = A'S$;

d) show that there exists a matrix $M \in \Gamma'$ such that the bottom row of AM is of the form $(0, *)$.

Deduce that $AM \in \Gamma$ and then that $\Gamma = \Gamma'$.