Univerität Basel

Herbsemester 2012

Master course A. Surroca - L. Paladino

Some topics on modular functions, elliptic functions and transcendence theory

Sheet of exercises n.7

7.1. Let $\Lambda = \mathbb{Z}_{\omega_1} \oplus \mathbb{Z}_{\omega_2}$ be a complex lattice. Let f be a meromorphic function. Prove that f is Λ-periodic if and only if $f(z+\omega_1) = f(z) = f(z+\omega_2)$, for all $z \in \mathbb{C}$.

7.2. Let f be a meromorphic function. Prove that if f is an elliptic function, then f'/f is an elliptic function.

7.3. Let f be an elliptic function. Prove that the poles and the zeroes of f are simple poles of f'/f and $\mathrm{Res}_{z_i}(f'/f) = \mathrm{ord}_{z_i}(f) = m_i$, with $m_i > 0$ if z_i is a zero and $m_i < 0$ if z_i is a pole.